Universe Scientific Publishing

Screening and Evaluation of Operational Effectiveness Indicators for Air Defense Missiles Based on Improved PCA

Peng Zhang, Ke Feng, Jiancheng Gong, Congcong Gong, Kai Zhao

College of Field Operations Engineering, Army Engineering University of PLA, Nanjing 210000, China.

Abstract: With the significant role played by air defense missile weapon systems in various local wars and armed conflicts, the significance of operational effectiveness evaluation has become increasingly important. This article determines the operational efficiency index system model of the air defense missile weapon system by constructing the operational environment of the air defense missile weapon system. The improved principal component analysis(PCA) method and MATLAB programming are applied to reduce the dimensionality and decorrelation of the operational performance indicators of the air defense missile weapon system's operational environment, and the optimized indicator system model of the operational efficiency indicators of the air defense missile weapon system is output. At the same time, the weight value is also closer to the actual combat situation, to achieve an accurate and express evaluation of combat effectiveness. By analyzing a case of evaluating the operational effectiveness of a certain type of air defense missile weapon system, key indicators were selected, and the effectiveness of the model in screening the operational effectiveness evaluation indicators of air defense missile weapon systems was verified. This provides a basis or reference for the subsequent optimization of air defense missile weapon system schemes and the evaluation of operational effectiveness and has certain promoting significance.

Keywords: Improved Principal Component Analysis Method; Air Defense Missiles; MATLAB; Indicator Screening; Operational Effectiveness Evaluation

1. Introduction

The operational efficiency indicators of air defense missile weapon systems are important measures to measure the effectiveness of air defense operations. The combat effectiveness not only reflects the performance level of the air defense weapon system but also reflects the strength of the air defense system's ability to complete tasks[1]. To accurately evaluate the combat effectiveness of air defense systems, it is necessary to start from reality and use scientific evaluation methods to evaluate the combat effectiveness of the system[2].

When establishing a combat effectiveness model for air defense missile weapon systems, due to the numerous characteristic indicators that affect the system's combat effectiveness, it is impossible to establish a performance indicator system model that includes all the factors that affect combat effectiveness. Therefore, selecting as few characteristic indicators or indicator groups as possible is necessary to establish an evaluation model[3]. Usually, multiple performance indicators are simplified into a few key performance indicators, and the essence is to select some indicators to reflect all indicator information. It will inevitably lead to the loss of some reduced indicators information[4]. This article uses an improved principal component analysis method to reduce the dimensionality and decorrelation of the operational effectiveness indicators of air defense missile weapon systems. This method contains as much information as possible about all indicators while minimizing the number of indicators, providing a basis and reference for selecting operational plans and evaluating the operational effectiveness of subsequent air defense missile weapon systems. By improving the principal component analysis method, we can more accurately evaluate the operational effectiveness of air defense missile weapon systems, thereby laying a favorable foundation for the victory of air defense operations.

2. Operational Environment of Air Defense Missile Weapon System

Aerial attacks are an important form of modern warfare and the most realistic and urgent threat facing a country. Air defense operations are a special type of military operation, whose core goal is to counter enemy air force attacks and raids through effective observation[6], positioning, decision-making, and action (OODA) cycles[7], and protect important targets and strategic resources of the enemy. In this process, various links are interconnected, forming a complete combat cycle[5].

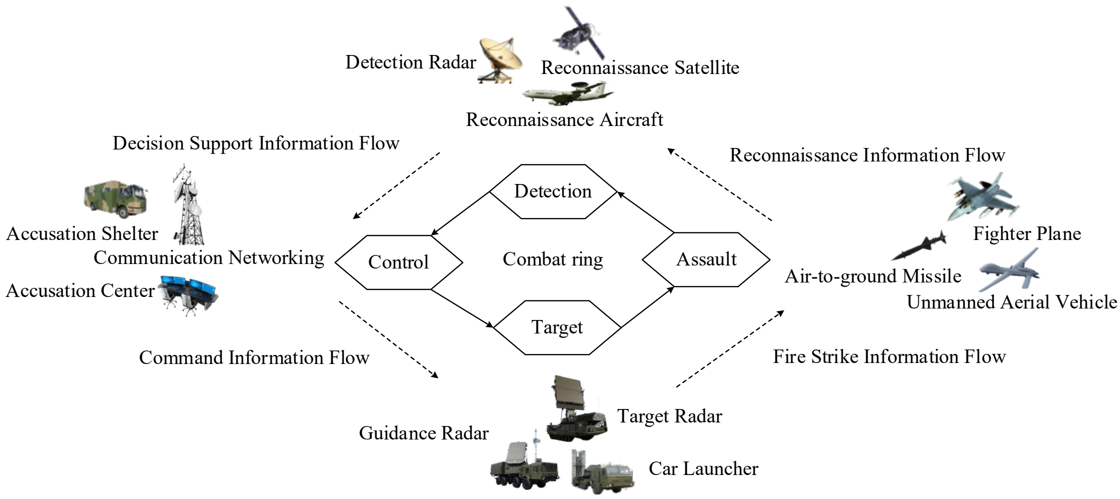

Figure 1 Representation of the Combat Loop

To cope with the complexity of air raid operations, in actual air defense combat tasks, multiple reconnaissance and decision-making entities are generally used to jointly complete[8]. Reconnaissance nodes represented by early warning radar and detection radar first obtain enemy invasion target information, and quickly transmit target data to command nodes represented by the command center; Command and control nodes form operational commands and plans after conducting threat assessment and intelligence information fusion[9]; Attack nodes represented by guided radars and missile launchers receive combat commands and effectively strike enemy targets. The reconnaissance phase includes multiple nodes such as satellite reconnaissance, reconnaissance aircraft reconnaissance, and radar detection, and there is information sharing between nodes; Command nodes also include the operational collaboration of multiple command entities; The attack class node includes multiple firepower units such as guidance radar and missile launch lights. Therefore, the most complex process in the combat process involves information sharing among multiple reconnaissance entities and collaborative command among multiple decision-making entities, and multiple combat rings together form a complex air defense combat network system . The representation of the air defense combat ring is shown in Figure 1.

3. Construction of Operational Effectiveness Index System for Air Defense Missile Weapon Systems

By analyzing the operational indicators of each node in the combat loop of the air defense missile weapon system, a combat effectiveness indicator model is established. Firstly, the combat technical indicators of each node in the combat loop must be described, which is the combat technical indicators related to the independent combat capabilities of each firepower combat unit in the air defense missile weapon system.

3.1 Reconnaissance Nodes

Reconnaissance nodes in air defense operations use reconnaissance and early warning systems to detect, locate, and identify enemy invading targets, and transmit target information to other nodes through data links. Therefore, considering the characteristics of the target and the impact of the combat environment, the detection capability X1 of the constructed reconnaissance node mainly includes radar detection probability x1, radar recognition probability x2, radar guidance probability x3, maximum detection distance x4 of the target, and terrain obscuration x5. Among them, the radar detection probability x1, identification probability x2, and guidance probability x3 are the inherent performance indicators of the air defense missile weapon system.

3.1.1 Maximum Detection Distance of the Target x4

$$x_4 = \frac{R_p}{200} \quad (1)$$

In the equation, Rp is the farthest detection distance of the radar detection system for high-altitude targets with an altitude of Hp (km), and 200 km is the typical value of the detection distance for aircraft targets.

3.1.2 Terrain Masking

$$x_4 = \frac{K_1 \cdot R_M}{50} \quad (2)$$

In the equation, RM (km) is the detection distance of low-altitude missile targets, 50 km is the typical value of the detection distance of low-altitude targets, and K1 is the correct coefficient of the obscuration angle, with the value of:

$$K_1 = \begin{cases} 1 - \frac{\theta_{ZB} \cdot R_P}{H_P}, & \frac{\theta_{ZB} \cdot R_P}{H_P} < 1 \\ 0, & \frac{\theta_{ZB} \cdot R_P}{H_P} \geq 1 \end{cases} \quad (3)$$

In the equation, θZB (Arc) is the main defensive sector shielding angle.

3.2 Accusation Node

In air defense combat tasks, command and control nodes integrate the target element information, battlefield environment, and intelligence information obtained by reconnaissance nodes, analyze the current battlefield situation, judge the enemy's invasion intention, make decisions, and issue combat orders . As the core equipment of air defense combat command, the command and control system are a key link to ensure accurate processing of combat information and execution of combat commands. Therefore, the command and control capability X2 mainly includes channel capacity x6, information processing delay x7, battlefield situation assessment ability x8, auxiliary decision-making ability x9, and decision response time x10.

Channel capacity x6 and information processing delay x7 are inherent performance indicators of air defense missile weapon systems; battlefield situation assessment capability x8 refers to the ability of air defense weapon systems to analyze and predict information acquisition, while auxiliary decision-making capability x9 refers to the ability to assist commanders in generating combat plans based on information acquisition and situational awareness, which is rated by experts; decision response time x10 is the average time taken to generate auxiliary decision plans and is also an inherent indicator of the system.

3.3 Attack Class Nodes

The main function of attack nodes is to guide radar and other specific combat operations based on combat commands, achieve the destruction of enemy invasion targets, and achieve combat objectives . The factors that affect the attack capability of air defense missile weapon systems mainly include shooting ability X3, anti-interference ability X4, missile characteristics X5, survivability X6, and so on.

3.3.1 Shooting Ability X3

$$X_3 = \frac{10 \cdot K_1}{x_{11}} \cdot x_{12} \cdot x_{13} \cdot x_{14} \quad (4)$$

In the equation, x11 is the weapon reaction time; x12 is the kill zone indicator group, x13 is the firepower intensity indicator group, x14 is the single shot kill probability, and x15 is the correction coefficient.

（1）The mathematical model of kill zone indicator group A is:

$$x_{12} = \left(\frac{x_{121}}{3} + \frac{0.5}{x_{122}}\right) \cdot \left(\frac{x_{123}}{4} + \frac{0.5}{x_{124}}\right) \cdot \frac{x_{125}}{45} \cdot \frac{x_{126}}{45} \quad (5)$$

In the equation, x121, x122, x123, x124, x125, and x126 refer to the high altitude, low altitude, far range, near range, maximum elevation angle, and maximum route angle of the kill zone.

（2）The mathematical model of firepower intensity index group x13 is:

$$x_{13} = x_{131} \cdot \frac{x_{132} \cdot x_{133}}{x_{134}} \cdot \frac{15}{x_{135}} \qquad (6)$$

In the equation, x131 is the target capacity, x132 is the number of launch devices, x133 is the number of missile joint installations, x134 is the number of single target projectiles, and x135 is the missile loading time.

3.3.2 Anti-interference Ability X4

The anti-interference performance of search, tracking, and guidance equipment in air defense missile weapon systems is an important technical indicator for dealing with complex electronic warfare environments. Electromagnetic information is transmitted and processed in multiple stages at each node, and any distortion of information in any stage will reduce the operational efficiency of the air defense missile weapon system, and even damage its normal operation[4]. Therefore, the anti-interference ability is quantified and decomposed into: the anti-interference ability of the guidance radar system x15, the anti-interference ability of the command line x16, and the anti-interference ability of the communication system x17. The values are given through relevant combat examples and expert scoring.

3.3.3 Missile Characteristics X5

$$X_5 = (\frac{x_{18}}{10}) \cdot x_{20} \cdot (\frac{x_{19}}{5}) \qquad (7)$$

According to the concept of missile characteristics, the larger the available overload x18, engine working time x19, and maximum speed x20 of the missile, the greater the missile performance index.

3.3.4 Survival Capability X6

Survivability refers to the ability of air defense missile weapon systems to continuously and effectively carry out combat tasks under enemy attack. This indicator includes anti-reconnaissance capability x21 (including position camouflage, behavior camouflage, and decoy capability), anti-damage capability x22, mobility capability x23, and system availability x24. The indicators of anti-reconnaissance capability and anti-damage capability are scored by experts.

（1）The mathematical model of mobility x23 is:

$$x_{23} = (\frac{15}{x_{231}} + \frac{6}{x_{232}}) \cdot (\frac{x_{233}}{40}) \cdot (\frac{x_{234}}{500}) \qquad (8)$$

By comparing the operational indicators of typical air defense missile weapon systems, a universal mobility mathematical model is established, where x231 is the deployment time of the weapon system, x232 is the withdrawal time of the weapon system, x233 is the maximum marching speed, and x234 is the maximum marching distance.

（2）The mathematical model of system availability x24 is:

$$x_{24} = \frac{x_{241}}{x_{241} + x_{242}} \qquad (9)$$

The availability of air defense missile weapon systems is a comprehensive reflection of system, reliability, maintainability, and supportability elements. In the formula, x241 is the average time between failures of the system, and x242 is the average repair time of the system.

3.4 Target Class Node

The target nodes mainly refer to the weapons and equipment used by the enemy to attack and damage our airspace. Abstracting enemy attack targets as nodes in our air defense combat environment is the core of building an air defense combat environment[5]. The selection of indicators for target nodes mainly considers the operational efficiency indicators that can effectively affect the effectiveness of our attack by enemy targets.

The mathematical model of target characteristic X7 is:

$$X_7 = x_{25} \cdot x_{26} \cdot (\frac{K_2}{x_{27}}) \cdot (\frac{x_{28}}{340}) \quad (10)$$

In the formula, the more types of targets x25 that can be intercepted, the stronger the maneuvering overload capacity x26, the smaller the radar cross-section x27, and the greater the maximum speed x28, indicating that the air defense missile weapon system needs to intercept the stronger the targeting ability.

By analyzing the mathematical models of operational capability indicators for each node of the air defense combat environment mentioned above, a combat effectiveness indicator system for the air defense missile weapon system is established, as shown in Figure 2 .

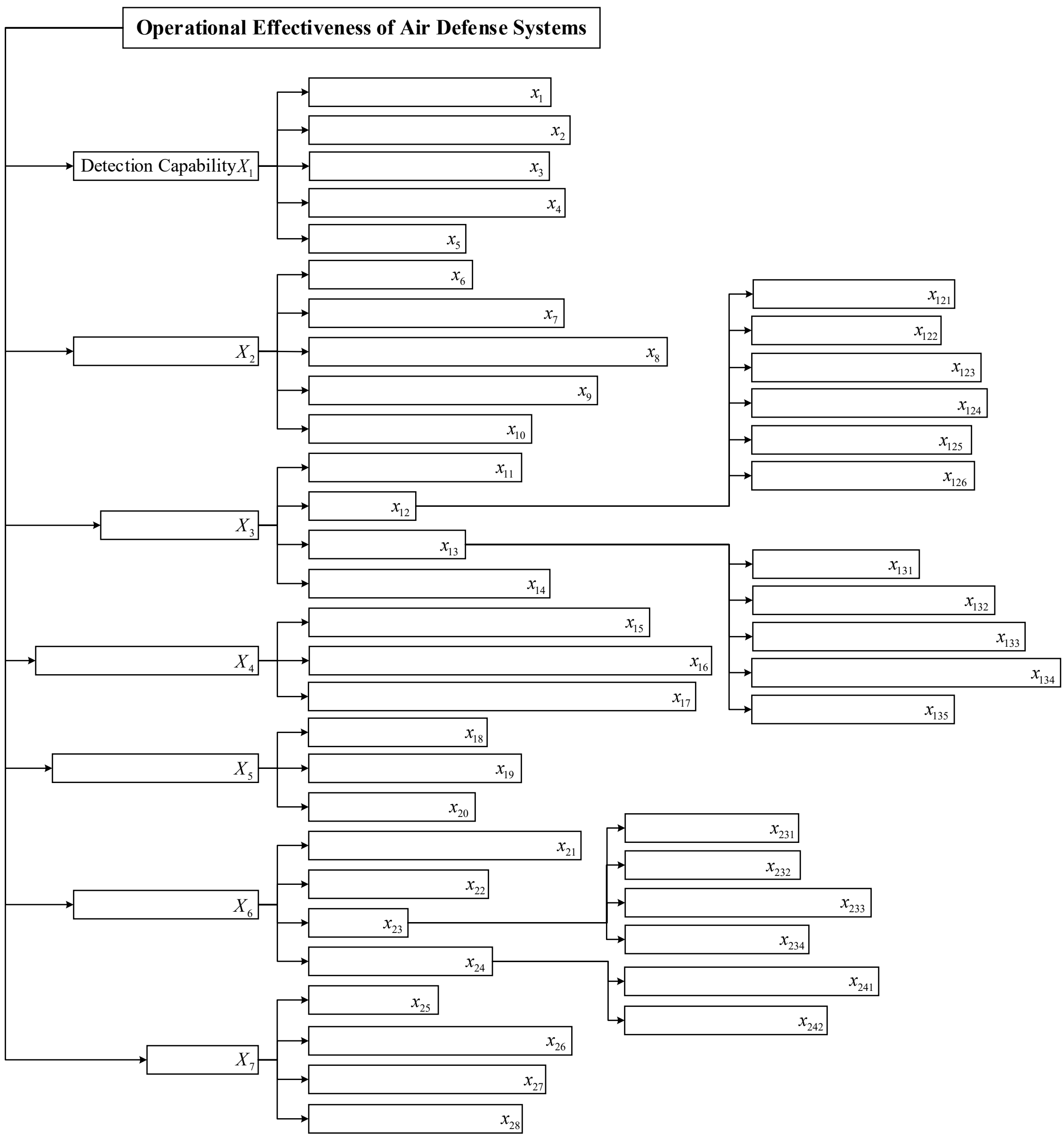

Figure 2 Operational Effectiveness Index System of Air Defense Missile Weapon System

The air defense missile weapon system is a typical complex system that integrates reconnaissance and early warning, firepower interception, command and control, network communication, electromagnetic countermeasures, and other systems. The interaction between each component system is frequent, and the number of efficiency indicators is large, Moreover, there is a strong correlation between indicators, which can cause information overlap. If the calculation system of all combat effectiveness indicators is directly applied, the calculation of combat effectiveness will not only be cumbersome and complex but also lead to a lack of credibility and accuracy in the effectiveness evaluation results due to the influence of some irrelevant indicators. Therefore, it is necessary to start from the characteristics of system composition and provide a comprehensive method for performance indicators from the perspective of system composition. To reduce the impact of the correlation between indicators, this article adopts an improved principal component analysis method.

4. Improvement Ideas for Principal Component Analysis Method

Although traditional principal component analysis can eliminate the correlation between indicators, using standardized processing of original indicators in the calculation process can lead to the loss of original information, and the weight of principal components completely depends on mathematical calculations. When applied in air defense operations, targeted adjustments cannot be made based on the actual battlefield situation, which is detached from the battlefield environment and inconsistent with the actual combat situation[3]. To reduce the correlation between the effectiveness indicators of air defense missile weapon systems, artificial weighting should be carried out promptly based on the different focuses of different battlefield environments, to make the evaluation results of combat effectiveness more reasonable. Propose ideas for improving principal component analysis from the following two aspects.

（1）The air defense missile weapon system contains numerous weapon units with independent combat functions, and their combat capabilities vary. Therefore, the operational efficiency indicators of each air defense weapon unit are not only not identical in terms of dimensions and orders of magnitude, but also more complex in quantity, with significant differences. If all operational indicator data is directly applied to combat effectiveness evaluation, the evaluation results may sometimes lean towards support indicators with larger orders of magnitude and significant differences, resulting in unreliable evaluation results. Moreover, the covariance matrix is highly susceptible to its influence. Although the use of standardized methods for data processing eliminates the impact of dimensionality and order of magnitude, it also eliminates differences in the degree of variation of various performance indicators, resulting in partial information loss. Therefore, it is recommended to improve the standardization method of principal component analysis by normalizing the optimal value of performance indicators and then standardizing them.

（2）Operational efficiency is the ability of an air defense missile weapon system to strike enemy targets and complete combat tasks. The various operational efficiency indicators of the system reflect their operational capabilities differently, and their effects are also different, but they all have different important meanings. In the traditional principal component analysis method, weight values are obtained by calculating indicator values. The objective weight values are not closely related to the actual combat situation, and there is a hidden danger of disconnection from the actual situation, resulting in imprecise evaluation results of combat effectiveness. Therefore, on this basis, a combination of scoring by command and staff personnel and the entropy weighting method is adopted to improve the shortcomings of relying solely on numerical calculation to obtain weights, making the combat effectiveness evaluation results more closely related to the battlefield situation at that time, with higher accuracy and credibility.

5. Basic Principles of Improving Principal Component Analysis

（1） Pre-processing of evaluation indicators.

Based on the constructed combat effectiveness index system of the air defense missile weapon system, an evaluation index matrix X is constructed, which includes n sets of raw data, and Each group of data includes p indicators X1, X2, …, Xp:

$$X = \begin{bmatrix} X_1 & X_2 & \cdots & X_p \end{bmatrix} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1p} \\ x_{21} & x_{22} & \cdots & x_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{np} \end{bmatrix} \quad (11)$$

The dimensions of various indicators of the air defense missile weapon system vary, and it is necessary to normalize the original indicator data xij into standardized indicator variables. The method of using the optimal value for normalization and then standardizing will not lose any information on the differences between indicators. The optimal value of the j-th performance indicator is X-max, and the normalization process is as follows:

$$x_{ij}^{'} = \frac{x_{ij}}{x_{j_{\max}}} \qquad (12)$$

The average value of the j-th performance indicator is:

$$\overline{x}_j = \frac{\sum_{i=1}^{n} x_{ij}}{n x_{j_{\max}}} \qquad (13)$$

Standardize the raw data:

$$x_{ij}^{*} = \frac{(x_{ij} - \overline{x}_j)}{\sqrt{Var(x_j)}}, \quad (i = 1,2,\cdots,n; j = 1,2,\cdots,p) \qquad (14)$$

其中，$Var(x_j) = \frac{1}{n-1}\sum_{i=1}^{n}(x_{ij} - \overline{x}_j)^2$ 。

Among them, $Var(x_j) = \frac{1}{n-1}\sum_{i=1}^{n}(x_{ij} - \overline{x}_j)^2$

（2）Indicator weighting.

Using expert scoring and entropy weighting methods to give the weights W = (w1, w2, …, wp) of p indicators, Among them $\sum_{i=1}^{p} w_i = 1$, the weighted matrix R is obtained:

$$R = \begin{bmatrix} x^{*}_{11} & x^{*}_{12} & \cdots & x^{*}_{1p} \\ x^{*}_{21} & x^{*}_{22} & \cdots & x^{*}_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ x^{*}_{n1} & x^{*}_{n2} & \cdots & x^{*}_{np} \end{bmatrix} \cdot \left[w_1, w_2, \cdots, w_p \right] \qquad (15)$$

（3）Correlation coefficient matrix.

For the convenience of symbol representation, the original matrix is still represented by the symbols of the original matrix after standardization. The method for calculating the element values of the coefficient matrix provided later involves xi and xj, which are not the original matrix, but the standardized matrix elements, only maintaining the original symbols.

$$R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1p} \\ r_{21} & r_{22} & \cdots & r_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{np} \end{bmatrix} \qquad (16)$$

Among them, $r_{ij} = Cov(x_i, x_j) = \frac{\sum_{k=1}^{n}(x_i - \overline{x}_i)(x_j - \overline{x}_j)}{n-1}, n > 1$

（4）Calculate the eigenvalues (λ1, λ2, …, λp) and corresponding eigenvectors:

$$a_i = (a_{i1}, a_{i2}, \cdots, a_{ip}), i = 1,2,\cdots,p$$

（5）Screening of principal components.

The variance of each principal component is decreasing, and the information contained is also decreasing. Therefore, when using this method in practice, not all principal components are selected, but the first k principal components are selected based on the cumulative contribution of each principal component . Combined with the performance characteristics of air defense missile weapon systems, the principal components with a cumulative contribution rate of 95% are selected, namely:

$$\alpha_i = \frac{\lambda_k}{\sum_{i=1}^{p} \lambda_i}, \quad (i = 1, 2, \cdots, p) \qquad (17)$$

6. Application of Principal Component Analysis in Missile Performance Evaluation

Eight types of anti-aircraft missile weapon systems, including “Dr2”, “Wa5”, “Gi1”, “GaA”, “Gr1”, “BoM”, “GaA1”, and “Tr0”, were selected as evaluation samples . Performance indicators include flight speed, maximum effective range, radar launch area, cruise altitude, anti-interference ability, system response time, environmental adaptability, and warhead power The specific data of terminal maneuvers, launch sector angles, and single shot hit probabilities are shown in Table 1. It should be noted that the collected data has been processed, with a focus on explaining and verifying the effectiveness of the method.

To verify the feasibility of using the improved principal component analysis method to screen the operational efficiency evaluation indicators of air defense missile weapon systems, the improved principal component algorithm was edited using Matlab to achieve the screening of operational efficiency indicators and the evaluation of operational efficiency of air defense missile weapon systems[4].

To overcome the problem of completely relying on numerical calculation for determining the weight of indicator values, combined with the application of battlefield environment and reality, the ANP commander and other expert scoring methods combined with the entropy weight method were used to determining the weight, The weights of 41 performance indicators are 0.05, 0.08, 0.10, 0.03, 0.01, 0.05, 0.04, 0.004, 0.01, 0.003, 0.06, 0.03, 0.03, 0.03, 0.03, 0.03, 0.02, 0.02, 0.01, 0.01, 0.005, 0.19, 0.006, 0.006, 0.008, 0.01, 0.01, 0.004, 0.001, 0.003, 0.002, 0.002, 0.001, 0.001, 0.001, 0.001, 0.001, 0.003, 0.038, 0.023.

Table 1 Performance Parameters of Air Defense Missile Weapon System

MODEL / INDEX	Dr2	Wa5	Gi1	GaA	Gr1	BoM	GaA1	Tr0
Radar Detection Probability	0.790	0.830	0.640	0.910	0.900	0.890	0.880	0.780
Radar Recognition Probability	0.810	0.870	0.710	0.650	0.790	0.710	0.760	0.650
Radar Guidance Probability	0.870	0.850	0.870	0.810	0.890	0.880	0.810	0.710
Maximum Detection Distance	75.00	175.0	100.0	150.0	220.0	230.0	150.0	200.0
Terrain Masking	0.630	1.430	0.720	1.350	0.210	0.420	1.250	1.130
Channel Capacity	441.0	404.0	465.0	412.0	389.0	444.0	409.0	426.0
Information Processing Delay	0.627	0.953	0.853	0.047	0.953	0.752	0.064	0.811
Battlefield Situation Assessment Capability	0.056	0.047	0.225	0.047	0.953	0.235	0.089	0.622
Assistant Decision-making Ability	0.083	0.047	0.354	0.232	0.953	0.445	0.062	0.453
Decision Response Time	17.00	24.00	25.00	17.00	14.00	9.000	40.00	30.00
Weapon Reaction Time	0.047	0.218	0.828	0.933	0.953	0.038	0.641	0.512
High Boundary Height	50.00	30.00	35.00	75.00	25.00	25.00	35.00	27.00
Lower Bound Height	0.300	0.100	0.200	0.300	0.100	0.300	0.500	0.200
Distant Diagonal Distance	180.0	75.00	300.0	230.0	200.0	100.0	160.0	120.0
Proximal Oblique Distance	1.000	5.000	5.000	3.000	2.000	3.000	1.000	5.000
Maximum Height Angle	80.00	47.00	88.00	60.00	70.00	45.00	88.00	62.00
Maximum Airway Angle	75.00	30.00	70.00	45.00	60.00	45.00	74.00	32.00
Target Capacity	8.000	6.000	12.00	8.000	8.000	6.000	6.000	10.00
Number of Launch Devices	8.000	3.000	8.000	6.000	8.000	6.000	4.000	8.000
Number of Missile Installations	4.000	4.000	6.000	6.000	8.000	4.000	6.000	4.000

Number of Single Target Projectiles	3.000	4.000	5.000	3.000	4.000	4.000	2.000	2.000
Missile Loading Time	0.953	0.927	0.047	0.667	0.760	0.053	0.841	0.620
Single Shot Kill Probability	0.740	0.600	0.650	0.800	0.650	0.800	0.750	0.720
Command Line Anti-interference Ability	0.750	0.880	0.850	0.650	0.790	0.770	0.790	0.740
Communication System Anti-interference Ability	0.630	0.880	0.680	0.950	0.890	0.830	0.750	0.640
Guidance Radar Anti-interference Capability	0.820	0.950	0.830	0.850	0.820	0.810	0.840	0.660
Available Overload	8.000	4.000	8.000	7.000	8.000	5.000	8.000	7.000
Engine Operating Time	50.00	35.00	45.00	45.00	50.00	30.00	32.00	35.00
Maximum Speed	1.400	0.900	1.100	1.900	1.900	1.000	1.300	2.000
Anti Reconnaissance Capability	0.750	0.720	0.570	0.680	0.610	0.580	0.530	0.500
Damage Resistance	0.720	0.490	0.700	0.750	0.590	0.660	0.640	0.530
Deployment Time	25.00	20.00	18.00	22.00	20.00	21.00	16.00	18.00
Withdrawal Time	15.00	10.00	8.000	14.00	12.00	14.00	7.000	9.000
Maximum Marching Speed	60.00	55.00	45.00	50.00	50.00	55.00	60.00	50.00
Maximum Mileage	650.0	700.0	700.0	700.0	800.0	750.0	750.0	600.0
Mean Time Between Failures	110.0	67.00	65.00	140.0	65.00	150.0	80.00	160.0
Mean Time to Repair	3.000	0.600	1.000	2.200	0.700	2.500	1.000	3.000
Target Type	8.000	6.000	6.000	4.000	12.00	10.00	10.00	10.00
Target Maneuvering Overload	0.580	0.770	0.680	0.620	0.600	0.550	0.610	0.500
Target Radar Cross-section	0.200	1.000	0.500	1.000	0.250	0.500	0.500	0.300
Target Maximum Speed	1.500	2.000	2.200	1.800	1.200	1.000	1.500	1.500

Establish the original data matrix X as:

$$X = \begin{bmatrix} 0.79 & 0.83 & 0.64 & 0.91 & 0.90 & 0.89 & 0.88 & 0.78 \\ 0.81 & 0.87 & 0.71 & 0.65 & 0.79 & 0.71 & 0.76 & 0.65 \\ 0.87 & 0.85 & 0.87 & 0.81 & 0.89 & 0.88 & 0.81 & 0.71 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0.20 & 1.00 & 0.50 & 1.00 & 0.25 & 0.50 & 0.50 & 0.30 \\ 1.50 & 2.00 & 2.20 & 1.80 & 1.20 & 1.00 & 1.50 & 1.50 \end{bmatrix}$$

6.1 Matlab Implementation Based on Improved PCA Method

(1) Set X as the matrix of 41 indicators for 8 types of anti-aircraft missile weapon systems, and read the performance parameter indicator values in the data file. Normalize using the optimal values of each indicator, and store the results in the variable normalized_X, retaining three significant digits after the decimal point.

```
>>X = xlsread('data.xls','B2:AP9');
>>max_values = zeros(1, size(X, 2));
>>for column = 1:size(X, 2)
max_values(column) = max(X(:, column));
end
>>normalized_X = X ./ max_values;
>>normalized_X = round(normalized_X, 3);
```

（2）Standardize matrix X to obtain matrix SA. Based on the known weights, obtain the final value of the variable data matrix.

```
>>a=size(X,1);
>>b=size(X,2);
>>for i=1:b
```

```
SX(:,i)=(normalized_X(:,i)-mean(normalized_X(:,i)))/std(normalized_X(:,i));
end
>>W=xlsread('data.xls','B11:AP11');
>>data=SX.*W;
```

（3）Perform principal component analysis on variable data and decompose the results into coeff, score, d, and explained. These results are stored separately in variables. And obtain the values of eigenvalues, eigenvectors, and contribution rates.

```
>>[coeff, score, d, v, explained] = pca(data);
>>eigenvalues = diag(explained);
>>eigenvectors = coeff;
>>disp('Characteristic value：');
>>disp(eigenvalues);
>>disp('Feature vector：')
>>disp(eigenvectors);
>>explained = explained.';
>>disp('Contribution rates of 41 principal components：');
>>disp(explained);
```

6.2 Dimension Reduction of Indicator Quantity

Among them, calculate the eigenvectors eigenvectors corresponding to the correlation coefficient matrix, namely the coefficients of the principal components, as shown in the table below. By calculating the cumulative contribution rate and determining the principal components according to $\alpha i \geq 95\%$, a much smaller principal component score will be obtained than the p indicators. By eliminating overlapping information and reducing the dimension of the indicators, a comprehensive analysis of the obtained principal components can be conducted.

Table 2 Performance Parameters of Various Types of Air Defense Missile Weapon Systems

No.	PERFORMANCE INDEX	1	2	3	4	5	6	7
1	Radar Detection Probability	0.0983	0.1362	-0.4103	0.3393	-0.1340	-0.0815	0.0419
2	Radar Recognition Probability	-0.2663	0.4010	-0.3483	-0.0561	0.4440	-0.0083	0.1120
3	Radar Guidance Probability	-0.1571	0.8115	0.3040	0.1370	-0.3250	0.1037	-0.0655
4	Maximum Detection Distance	0.0078	-0.0333	-0.1999	0.0635	-0.3635	-0.1850	-0.2818
5	Terrain Masking	0.0016	-0.0239	-0.0224	0.0038	0.0358	0.0623	-0.0182
6	Channel Capacity	0.0440	0.0264	0.4121	-0.4578	-0.0383	0.1385	-0.1873
7	Information Processing Delay	-0.1371	0.0060	0.0108	-0.2301	-0.3299	-0.2238	0.1942
8	Battlefield Situation Assessment Capability	-0.0048	-0.0094	0.0007	0.0146	-0.0313	-0.0598	0.0172
9	Assistant Decision-making Ability	-0.0068	-0.0042	0.0195	0.0460	-0.1148	-0.1222	0.0351
10	Decision Response Time	-0.0015	-0.0151	-0.0020	0.0017	0.0352	-0.0012	-0.0521
11	Weapon Reaction Time	-0.0211	-0.2187	0.2892	0.6590	-0.0801	-0.0289	-0.1075
12	High Boundary Height	0.0694	0.0054	0.0683	0.1361	0.1405	0.3299	0.5343
13	Lower Bound Height	0.1045	0.0421	0.0141	0.0143	0.2866	0.0029	-0.4428
14	Distant Diagonal Distance	-0.0004	-0.0076	0.3247	0.1630	0.0489	0.0471	0.1287
15	Proximal Oblique Distance	-0.0571	-0.1657	0.0277	-0.1334	-0.2406	0.2285	-0.0808
16	Maximum Height Angle	-0.0094	-0.0060	0.2332	0.0807	0.3466	-0.1672	-0.1385
17	Maximum Airway Angle	0.0075	0.1254	0.2151	0.0698	0.2874	-0.1735	-0.1289
18	Target Capacity	-0.0170	-0.0938	0.1924	-0.0289	-0.0290	-0.0328	0.1535
19	Number of Launch Devices	0.0088	-0.0247	0.1538	-0.0285	-0.0725	-0.2086	0.3483
20	Number of Missile Installations	-0.0166	0.0112	0.0810	0.2365	-0.0273	-0.1001	-0.0696

21	Number of Single Target Projectiles	-0.0260	0.0374	0.0535	-0.0062	-0.0916	0.0516	-0.0216
22	Missile Loading Time	-0.0046	-0.0004	-0.0344	0.0188	0.0536	-0.0071	0.0760
23	Single Shot Kill Probability	0.9215	0.2149	-0.0099	-0.0415	-0.0137	-0.0029	0.0559
24	Command Line Anti-interference Ability	-0.0244	0.0083	0.0004	-0.0216	0.0039	0.0019	-0.1004
25	Communication System Anti-interference Ability	-0.0002	0.0132	-0.0280	0.0525	-0.0484	0.0460	-0.0022
26	Guidance Radar Anti-interference Capability	-0.0160	0.0412	-0.0167	0.0277	0.0173	0.0979	-0.0348
27	Available Overload	0.0083	-0.0099	0.0830	0.0537	0.0844	-0.1030	0.0556
28	Engine Operating Time	-0.0113	0.0174	0.0613	0.0480	0.0174	-0.0223	0.2580
29	Maximum Speed	0.0059	-0.0185	0.0001	0.0256	-0.0040	-0.0317	0.0744
30	Anti Reconnaissance Capability	-0.0010	0.0048	-0.0020	-0.0001	0.0029	0.0091	0.0234
31	Damage Resistance	0.0084	0.0105	0.0231	0.0064	0.0074	0.0122	0.0186
32	Deployment Time	0.0017	0.0097	-0.0007	-0.0049	-0.0023	0.0060	0.0578
33	Withdrawal Time	0.0037	0.0101	-0.0025	-0.0014	-0.0104	0.0024	0.0489
34	Maximum Marching Speed	0.0023	0.0090	-0.0130	-0.0056	0.0206	-0.0044	-0.0051
35	Maximum Mileage	-0.0006	0.0054	-0.0005	0.0081	-0.0046	-0.0028	-0.0156
36	Mean Time Between Failures	0.0039	-0.0026	-0.0022	-0.0047	-0.0050	-0.0005	0.0089
37	Mean Time to Repair	0.0034	-0.0012	0.0000	-0.0070	-0.0003	-0.0029	0.0169
38	Target Type	0.0000	0.0009	-0.0028	-0.0001	-0.0018	-0.0183	-0.0087
39	Target Maneuvering Overload	-0.0100	0.0052	-0.0010	0.0066	0.0062	0.0390	-0.0155
40	Target Radar Cross-section	-0.0057	-0.0281	-0.1337	0.1245	-0.0828	0.6752	-0.1379
41	Target Maximum Speed	-0.0497	-0.0738	0.1007	0.0015	0.1131	0.3087	0.0204

The greater the contribution rate of the principal component, the more information it contains about the original operational performance indicators. Through Matlab principal component analysis of the performance indicators of air defense missile weapon systems, seven principal components were obtained, and their contribution rates were 57.44%, 16.35%, 9.45%, 7.60%, 4.55%, 3.57%, and 1.03%, respectively. The greater the contribution rate, the stronger the information of the original variables contained in the principal component. Based on the combat effectiveness of the air defense missile weapon system, a cumulative contribution rate of over 95% is selected to ensure that the selected principal components contain sufficient information and achieve optimization and dimensionality reduction of performance indicators. Therefore, the top 5 principal components were selected.

The coefficients of 1, 6, 12, 13, 19, and 23 in the first principal component are the highest, with radar detection probability, channel capacity, high bound altitude, low bound altitude, number of transmitting devices, and single shot hit rate as the main performance indicators; The coefficients of 1, 2, 3, 13, 17, and 23 in the second principal component are the highest, with radar detection probability, radar recognition probability, radar guidance probability, low bound altitude, maximum airway angle, and single engine hit rate as the main performance indicators; The coefficients of 3, 6, 11, 14, 16, and 17 in the third principal component are the highest, with radar guidance probability, channel capacity, weapon response time, far range, maximum high low angle, and maximum route angle as the main performance indicators; The coefficients of 1, 3, 11, 14, 20, and 40 in the fourth principal component are the highest, with radar recognition probability, radar guidance probability, weapon response time, far range, missile joint number, and target radar cross-section as the main performance indicators; The coefficients of 2, 12, 13, 16, 17, and 41 in the fifth principal component are the highest, namely radar recognition probability, high bound altitude, low bound altitude, maximum high low angle, maximum route angle, and maximum target speed are the main performance indicators.

Therefore, after eliminating redundant and relevant information between operational efficiency indicators, the reduced dimensionality operational efficiency indicator system of the air defense missile weapon system is obtained as shown in Figure 3.

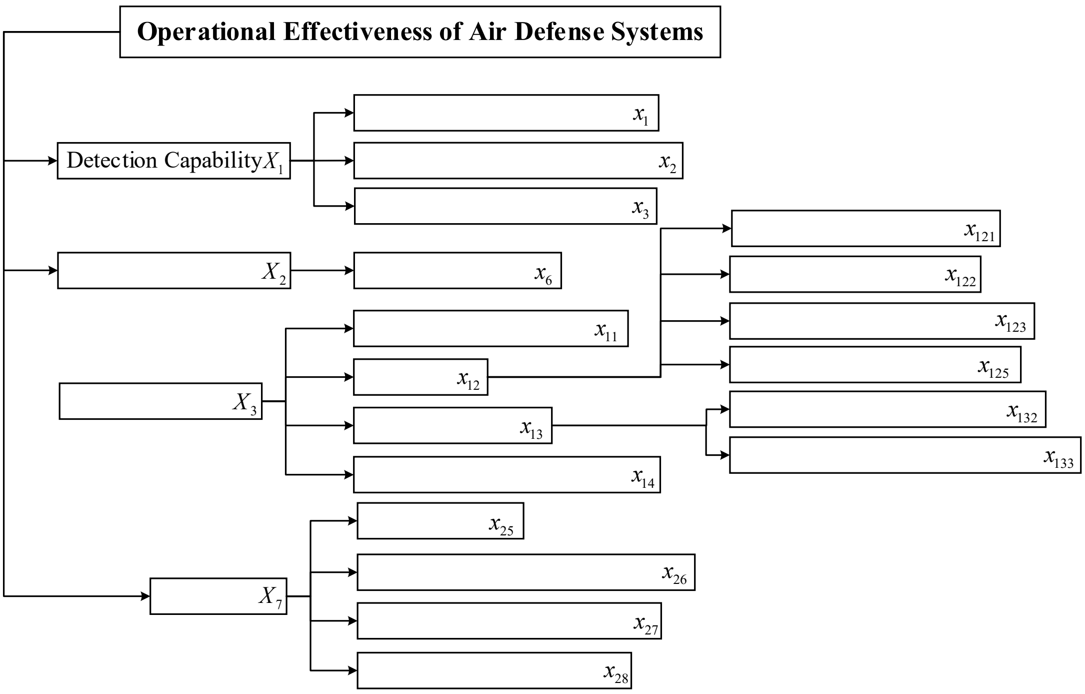

Figure 3 Operational Effectiveness Index System of Air Defense Missile Weapon System

The weight values of each indicator are:

Table 3 Performance Index Weight Values

No.	PERFORMANCE INDEX	1	2	3	4	5	Score	weight coefficient
1	Radar Detection Probability	√	√				81.39	14.17%
2	Radar Recognition Probability		√		√	√	20.90	3.64%
3	Radar Guidance Probability		√	√	√		33.40	5.82%
4	Channel Capacity	√		√			66.89	11.65%
5	Weapon Reaction Time			√	√		17.05	2.97%
6	High Boundary Height	√				√	61.99	10.80%
7	Lower Bound Height	√	√			√	78.34	13.64%
8	Distant Diagonal Distance			√	√		17.05	2.97%
9	Maximum Height Angle			√		√	14.00	2.44%
10	Maximum Airway Angle		√	√		√	30.35	5.29%
11	Number of Launch Devices	√					57.44	10.00%
12	Number of Missile Installations				√		9.45	1.65%
13	Single Shot Kill Probability	√	√				73.79	12.85%
14	Target Radar Cross-Section				√		7.60	1.32%
15	Target Maximum Speed					√	4.55	0.79%

6.3 Operational Effectiveness Evaluation

```
>> new_score=data*eigenvectors;
>> for i=1:a
total_score(i,1)=sum(new_score(i,:));
total_score(i,2)=i;
end
>> result_report=[new_score,total_score];
```

```
>> result_report=sortrows(result_report,-8);
```

Based on normalized and standardized performance index data of air defense missile weapon systems, the expressions of principal components are carried in for each derived model. The first 7 columns represent the scores of each principal component, the 8th column represents the total score of each principal component, and the 9th column represents the number of air defense missiles. The operational efficiency of each missile weapon system is sorted in descending order of total score.

$$result\_report = \begin{bmatrix} 0.2532 & -0.0388 & 0.0087 & 0.0950 & -0.0156 & 0.0720 & 0.0239 & 0.3984 & 4 \\ 0.0417 & 0.1162 & 0.0252 & -0.0699 & 0.0732 & -0.0192 & 0.0408 & 0.2080 & 1 \\ 0.1072 & 0.0043 & -0.0321 & 0.0507 & 0.0909 & -0.0182 & -0.0444 & 0.1584 & 7 \\ 0.2100 & 0.1191 & -0.0270 & -0.0716 & -0.0843 & -0.0060 & -0.0228 & 0.1175 & 6 \\ -0.1684 & -0.0326 & 0.1773 & -0.0234 & -0.0087 & 0.0264 & -0.0175 & -0.0469 & 3 \\ -0.1865 & 0.0509 & -0.0042 & 0.1092 & -0.0451 & -0.0741 & 0.0122 & -0.1377 & 5 \\ 0.0655 & -0.2247 & -0.0329 & -0.0622 & -0.0113 & -0.0450 & 0.0093 & -0.3012 & 8 \\ -0.3226 & -0.1150 & -0.1150 & -0.0278 & 0.0008 & 0.0640 & -0.0016 & -0.3965 & 2 \end{bmatrix}$$

According to the calculation results, the combat effectiveness of the Type 4 "GaA" anti-aircraft missile weapon system is the strongest, while the combat effectiveness of the Type 2 "Wa5" anti-aircraft missile weapon system is the weakest. By performing dimensionality reduction and decorrelation processing on the operational performance indicators of air defense missile weapon systems, simulation efficiency has been improved for the application of subsequent performance indicators or the evaluation of operational effectiveness, avoiding unreasonable weights given due to the subjective decisiveness of expert experience and the impracticality of relying on objective data, making the evaluation results more objective.

7. Conclusion

This article proposes to improve the dimensionless method of PCA by constructing an operational effectiveness index system for air defense missile weapon systems and to compensate for the shortcomings of PCA solely relying on calculation to determine weights through subjective and objective weighting, making the evaluation results of the improved PCA more reliable.

As a typical complex system in the combat effectiveness evaluation process of air defense missile weapon systems, there is inevitably a large amount of high-dimensional data information in the combat effectiveness evaluation process. The improved principal component analysis method is used to reduce the dimensionality and decorrelation of high-dimensional data information. By reducing the dimensionality and extracting the main feature indicators that are easier to understand and contain stronger information, the impact of dimension explosion and data correlation on the combat effectiveness evaluation of air defense missile weapon systems in the combat environment is overcome, And it has accelerated the processing speed of system analysis and preserved the data information of the original variables to the greatest extent, laying a stable and fast foundation for the next step of data processing and system analysis, achieving the purpose of effectively evaluating the combat effectiveness of air defense missile weapon systems.

References

[1] Guo Xiaochuan, Liu Xinchang, Chen Guiming, et al. Missile weapon system-of-systems optimization method based on information entropy[C]//2016 International Conference on Computer, Information and Telecommunication Systems (CITS): IEEE, 2016: 1-5.

[2] Ji Hongquan, Hou Qingsen, Wu Dehao. Modified performance-enhanced PCA for incipient fault detection of dynamic industrial processes[J]. Journal of Process Control, 2023, 131: 103107.

[3] Ma Qingyue. Research on Comprehensive Evaluation Techniques for Weapon System of Systems' Operational Effectiveness[D]. Harbin Institute of Technology, 2015

[4] Qiao Rong, Zhou Feng. Combat Effectiveness Evaluation of Surface-to-Air Missile Weapon Systems Based on PCA-BP Neural Networks[J]. Military Operations Research and Systems Engineering, 2020, 34(04): 38-43, 67.

[5] Zhao Danling, Tan Yuejin, Li Jichao, et al. Armament System of Systems Contribution Evaluation Based on Operation Loop[J]. Systems Engineering and Electronics, 2017, 39(10): 2239-2247.

[6] Ozgur Tuncer, Hakan Ali Cirpan. Adaptive fuzzy based threat evaluation method for air and missile defense systems[J]. Information Sciences, 2023, 643: 119191.

[7] Wan Zhongyun, Li Yonggang, Zhang Zhizhong. Modeling and Analysis of Integrated Combat Network System Based on VOODAC[C]//Proceedings of the International Conference on Industrial Control Network and System Engineering Research: ACM, 2019.

[8] Han Qi, Pang Bo, Li Sen, et al. Evaluation method and optimization strategies of resilience for air & space defense system of systems based on kill network theory and improved self-information quantity[J]. Defence Technology, 2023, 21(03): 219-239.

[9] Xiong Peisen, Liu Hu, Tian Yongliang. Mission Effectiveness Evaluation of Manned/Unmanned Aerial Team based on OODA and Agent-Based Simulation[C]//Proceedings of 2019 3rd International Conference on Artificial Intelligence and Virtual Reality (AIVR 2019): ACM, 2019: 14-20.

[10] Yang Weisheng, Wang Yu, Yang Yang, et al. Combat Network Effectiveness Evaluation Under Different Node Attack Strategies Based on Operation Loop[J]. Engineering and Electronics, 2021, 43(11): 3220-3228.

[11] Danling ZHAO, Yajie DOU, Qingsong ZHAO, et al. The method to evaluate the command and control effectiveness of operational system under uncertain threat situation[C]// 28th Chinese Control and Decision Conference(CCDC), 2016: 787-792.

[12] Quentin Voortman, Alexander Yu. Pogromsky, Alexey S. Matveev, et al. Consensus in networks of dynamical systems with limited communication capacity[J]. Automatica, 2022, 145: 110514.

[13] Wang Jun, Zhaojie, Shao Lei, et al. System Effectiveness Evaluation Model of Ground to Air MissileBased on ADC Method[J]. Modern Defence Technology, 2015, 43(06): 13-20.

[14] Wang Jun, Zhao Jie, Li Jiong, et al. Research on numerical model of ground-to-air missile kill zone[J]. Systems Engineering Society of China, 2014, 34(12): 3260-3267.

[15] Zhi Hongxin, Zhao Peng, Li Zhong, et al. A Weapon-target Assignment in Air-defense Operations Based on Shooting Probability Constraint[J]. Acta Arnamentarh, 2022, 43(04): 952-959.

[16] Qi Zhangxing, Chen Yewei, Liu Yuan, □. Radar signal recognition based on deep convolutional neural network in complex electromagnetic environment[C]//2022 3rd China International SAR Symposium (CISS): IEEE, 2022: 1-5.

[17] Hong-hao Zuo, Xiao-ming Li. A novel calculation method of radar jamming cover area for ground to air radar countermeasures[Z], 2016.

[18] Peibei Ma, Jun Ji, Jiangbo Sui, et al. Tactic Technical Performance Analysis of Missile Based on Set Pair Method[C]//2021 International Conference on Computer Engineering and Application (ICCEA): IEEE, 2021.

[19] Zhao Yueqiang, An Shi, Mai Qiang, et al. Effectiveness Modeling of Air Defense Missile Weapon System Based on ADC Method[J]. Systems Engineering and Electronics, 2020, 42(09): 2003-2012.

[20] Gu Hui, Jiang Demao, Wang Guangyi, et al. Research on Effectiveness Evaluation Simulation Method of Surface-to-Air Missile Weapon Systems[C]//Proceedings of the 2014 International Conference on Electrical,Control and Automation, 2014: 349-357.

[21] Wang Jun, Zhou Lin, Lei Humin, et al. Medium and Far Range Ground-to-Air Missile System Effectiveness Evaluation Model[J]. Journal of System Simulation, 2010, 22(07): 1761-1768, 1772. [22]Aosong Liang, Yunpeng Hu, Guannan Li. The impact of improved PCA method based on anomaly detection on chiller sensor fault detection[J]. International Journal of Refrigeration, 2023, 155: 184-194.

[22] Dong Yuan, Nicholas Mancuso. SuSiE PCA: A Scalable Bayesian Variable Selection Technique for Principal Component Analysis[J]. Iscience, 2023: 108181.

[23] Yunsong Li, Jiahui Qu, Wenqian Dong, et al. Hyperspectral pansharpening via improved PCA approach and optimal weighted fusion strategy[J]. Neurocomputing, 2018, 315: 371-380.

[24] Kamila Zdybał, Elizabeth Armstrong, Alessandro Parente, et al. PCA fold: Python software to generate, analyze and improve PCA-derived low-dimensional manifolds[J]. Softwarex, 2020, 12: 100630.

[25]Amir Eshaghi Chaleshtori, Abdollah Aghaie. A novel bearing fault diagnosis approach using the Gaussian mixture model and the weighted principal component analysis[J]. Reliability Engineering & System Safety, 2024, 242: 109720.

[26] Fan junwei. The research of data extraction for remote sensing image information of NetCDF based on GDAL[D]. East China Institute of Technology, 20133

[27]Liu Yaosen, Zhang Zhouwei, Wang Yahong, et al. Overview of the Development of Russia's Missile Defense System[J]. Aerospace China, 2022, (06): 27-31.

[28]Bin Huang, Bong-Hwan Koh, Heung Soo Kim. PCA-based damage classification of delaminated smart composite structures using improved layer wise theory[J]. Computers & Structures, 2014, 141: 26-35.

[29]Libiao Bai, Chaopeng Song, Xinyu Zhou, et al. Assessing project portfolio risk via an enhanced GA-BPNN combined with PCA[J]. Engineering Applications of Artificial Intelligence, 2023, 126: 106779.