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RESEARCH ARTICLE

Reilly's gravity law adapted to finance

Guillermo Peña

Department of Economic Analysis and IEDIS, Universidad de Zaragoza, Zaragoza, Spain

*** Corresponding author:** Guillermo Peña, gpena@unizar.es

ABSTRACT

The present paper proposes new indicators that explain the pricing of the unitary pure interest rates and interests by adapting previous Reilly's Law (RL) models of urban economics and regional economics and Financial Gravity (FG) models for financial services. A panel dataset of 114 countries for 1967-2021 is handled, applying the OLS methodology. The results show the higher explanatory power of the adaptation of the FG models rather than the proposed adaptation of the RL ones.

Purpose: This paper tries to adapt previous Gravity Equations from Urban Economics to Finance and empirically checks whether they behave better than the recently found Financial Gravity (FG) models. For that, new indicators are provided jointly with their adaptations to Ordinary Least Square (OLS) methods. Additionally, an economic perspective of the bid-asks of financial products is provided.

Design/methodology/approach: OLS methods are used to check the desirability and accuracy of the new indicators. A panel dataset of 114 countries for 1967-2021 is handled, applying the OLS methodology.

Findings: The most adequate are the FG models after the empirical check, providing the desirable properties of those models. The results show the higher explanatory power of the adaptation of the FG models rather than the proposed adaptation of the RL ones.

Originality: This is the first paper, to the author's knowledge, on providing an adaptation of Reilly's Law from urban economics models to finance, also providing empirical evidence after adapting these models to linear expressions for applying OLS methods. We also provide evidence on the statistical equality between the unitary pure interest of loan and deposit interests, confirming the good properties of the FG models.

Keywords: pure interest; financial services; asset pricing; gravity models; cash flows

1. Introduction

Gravity Laws are essential in economics, overall in trade[1] (Ismail, 2021), but also for finance as recently shown[2,3] (López-Laborda and Peña, 2018, Peña, 2021). The present paper deals with the adaptation to finance of a well-known gravity law of urban economics and regional sciences, the Reilly's Gravity Law of economic activity between two cities and an intermediary one. So, the present paper proposes direct adaptations from regional sciences models, specifically from the Reilly's[4] (1931) Law model, to financial services, providing

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new indicators useful for pricing interests. The interlinks between spatial and temporal intermediation are explored in the present paper, which could be considered a starting point on financial geography for further developments and research. Additionally, this paper provides further explanatory formulations and derivations from the recently-proposed Financial Gravity (FG) models. An unbalanced panel of 114 countries for annual data from 1967 to 2021 will be handled and Ordinary Least Square (OLS) techniques will be applied. The results show that the most explanatory model is that adapted one from the FG models, instead of the one adapting Reilly's Gravity Law.

Next, the paper is divided as follows. Section 2 provides a brief summary of the conceptual aspects of the Reilly's Law, while Section 3 shows the essentials of the FG models. Section 4 reviews the literature in a brief way, Section 5 proposes the indicators and provides empirical evidence after showing the empirical strategy. Finally, Section 6 discusses the results and provides the concluding remarks.

2. Theoretical aspects for the Reilly's law

For joining regional with financial economics, a gravity model very often used in the literature of economic activity is the Reilly's[4] (1931) law of retail gravitation:

$$\frac{B_{ix}}{B_{jx}} = \left(\frac{N_i}{N_j}\right)^{1/\alpha} \left(\frac{D_{jx}}{D_{ix}}\right)^{\gamma} \tag{1}$$

being B_{ix} and B_{jx}, respectively, the economic activity which location *i/j* draws from an hypothetical intermediate location x, while the population is N. The variables α and γ are parameters. Finally, the distances between location x and locations i and j are, respectively, D_{ix} and D_{jx}. Nonetheless, there is no direct equivalent for financial services. The model closer to the previous proposal is developed by López-Laborda and Peña (2018), developing the Financial Gravity (FG) models[3] (Peña, 2021). These authors adapt the last advances by Poddar and English[5] (1997) in the calculation of the value added of the financial services. While the last authors consider the value added as a difference between the interest receipts IR and the pure interest ε for the value added of the interest receipts and the difference between this pure interest and the interest payments IP for the financial margin of the interest payments, the former authors consider the value added as a stable proportion ρ respect to the interest receipts and payments, reaching the same share for both kinds of interests.

3. The gravity models

This paper follows the Financial Gravity (FG) models proposed by Peña[3] (2021) and others. First, this section starts by considering financial services as a special case of financial transactions. Financial transactions can be considered in a general way, so according to Contreras and Contreras[6,7] (2015, 2018), a well-known formulation for financial transactions considering the supply (bid) and demand (ask) of any financial investment would be as follows:

$$
\begin{aligned}
p_a^i &= p_t^i\left(1+s_b^i\right) \\
p_v^i &= p_{t+n}^i = p_t^i\left(1+r_n^i\right)\left(1-s_a^i\right)
\end{aligned}
\tag{2}
$$

Where p_a^i is the purchase price of the financial product including the bid spread, s_b^i, p_t^i is the initial acquisition price (net spread), $p_v^i = p_{t+n}^i$ is the selling price, which is the price of the financial product at the time $t+n$, s_a^i is the ask spread and r_n^i is the rate of return.

One contribution of this paper is explicitly providing the following underlying assumptions for the proposals of Lopez-Laborda and Peña[2] (2018) for the first time, linking the public economics literature (the last authors) with private finance (the former authors). The next equations show the general assumptions that implicitly lie on banking services according to these last authors:

$$
\begin{aligned}
p_a^i &= p_v^i = \varepsilon \\
s_b^i &= s_a^i = \rho \\
p_t^i &= IP \\
p_t^i \cdot \left(1+r_n^i\right) &= IR
\end{aligned}
\tag{3}
$$

where ρ is the mobile ratio proposed by the last authors (modified quoted spread applied to the interest income that enables the allocation of the value added of the company among each financial service in order to be taxed with the rate of the Value Added Tax, VAT), IP are the interest payments, IR are the interest receipts and ε is the pure interest (interest without fees for risk). The explanation and intuition for these assumptions are the following ones. Financial services, and concretely, banking services, could be considered financial transactions where the bank purchases the deposit amount p_a^i by a price ε, which is the initial acquisition price p_t^i that the bank pays to the depositor customer (*IP*). This price is the price that Central Banks or the interbank market (they can be considered a kind of customers) pay to the bank for depositing the customer's deposits in there by a price ε paid by the financial markets or the central banks, which constitutes of the purchase price (p_a^i) for these customers, but minus the value added of the financial service (Financial Intermediation Service, FIS) in the form of deposit service generated by the bank, which is the acquisition price (*IP*) multiplied by the bid spread (s_b^i), which is the mobile ratio. So, in addition to "buying" deposits to customers, itself, the financial institution generates a value added (VA) equal to ε-*IP*, which is the same as the bid price ($s_b^i p_t^i$) reaching the ρIP amount.

Furthermore, the bank can also sell the FIS to a borrower as a loan by an acquisition price plus returns of $p_t^i\left(1+r_n^i\right)$ that constitutes the interest receipts (IR) for the bank selling price. For achieving the previous amount for the borrower, the bank purchases the money in the financial markets or to the central banks again (in this case, they can be considered a kind of wholesalers). Nonetheless, the selling price $p_v^i = p_{t+n}^i$ of this financial transaction for the wholesalers is again ε. The Value Added (VA) for the FIS in the loan is IR-ε, achieving a total value added of IR-IP for the whole financial service.

Finally, it is assumed that the marginal productivity of the financial services is the same and equals to the "bid" and "ask" spread. The main reason is that a company usually tries to equalize the mark-ups for all products. This is in contrast to equation (2), which shows a typical financial transaction of bid-ask where there is no intermediary and every business has their own, secret, marginal productivities, because there can be three financial institutions involved in the procedure of the financial transaction, but in the financial service only the same bank provides both loans and deposits, or the lonely-loan service, otherwise. This explanation is coherent with Peña[8] (2019), who consider that the intertemporal interest rate does not shift with financial intermediation, the capital income is the same and the difference is the value added or the price of the financial service, considered as financial consumption.

So, expression (4) could be formulated as follows:

$$\begin{aligned} \varepsilon &= IP\cdot\left(1+\rho\right) \\ \varepsilon &= IR\cdot\left(1-\rho\right) = IP\cdot\left(1+r_n^i\right)\cdot\left(1-\rho\right) \end{aligned} \tag{4}$$

where IR are the interest receipts. This formula, after some rearrangements, leads to the following system:

$$\left.\begin{aligned} \rho\cdot IR &= IR-\varepsilon \\ \rho\cdot IP &= \varepsilon-IP \end{aligned}\right\} \tag{5}$$

Finally, the former authors equalize their view for obtaining the value added to that from Poddar and English[5] (1997), achieving the next system of equations:

$$\left.\begin{aligned} \rho IR &= IR-\varepsilon \\ \rho IP &= \varepsilon-IP \end{aligned}\right\} \Rightarrow \rho = \frac{IR-IP}{IR+IP}; \varepsilon = \frac{2IRIP}{IR+IP}; \delta := \frac{2IRIP}{\left(IR+IP\right)^2} \tag{6}$$

Where the equations of the right side are easily derived from the previous system. In the next subsection two kinds of theoretical models will be developed, all of them at least slightly based on one of them but fundamentally based on the other. The expression (1) will lead to the Reilly's Law (RL) model of gravity for financial services, with two different specifications and the system (2) will enable to formulate two equivalent new proposals of FG models adapted for further studies, that is, the later estimation for empirical evidence of the next section.

4. Literature review of other recent gravity applications

Other applications of gravity models are found in the literature. For a historical review of the literature, see Capoani[9,10] (2021, 2023). Since the nineteenth century, there are applications to social, geographical and economic interactions, being more or less close to the original Newton's proposal. The Reilly's Law of Gravitational Retail, for instance, is recently used also for migration flows as in Guha et al.[11] (2023). In other applications, Zakharchenko and Oneshko.[12] (2022) use Newton's and Reilly's laws for explaining the choices determining consumers' preferences. Additionally, the shopping destination choice behavior (SDCB) of the customers can also be estimated by the Reilly's Law as recently in Biswas and Chattopadhyay.[13] (2024). Finally, trade is also a common issue to be dealt with Newton's and Reilly's Gravity Laws, see Jadhav and Ghosh.[14] (2024) for a very recent literature review in depth. In the whole literature, to the author's knowledge, there is no direct application of the Reilly's Law to financial services in contrast to the present paper.

5. Proposal of indicators and empirical illustration

5.1. Empirical strategy

The methodology to follow is as follows. We are going to consider two specifications at the beginning for considering the adaptation of the FG models of expression (1), and also two for the Reilly's Law models, respectively, RL (left) and RL2 (right) specifications from expression (1). The empirical specification for expression (4) would be, after rearranging, the following expressions explanatory of the unitary pure interest rate, hereafter denominated as 'FG rules' of deposit rates (R, and the indicator is *ratiodr*) and lending rates (r, and the indicator is *ratiolr*):

$$\begin{aligned} ratiodr &:= (1-\rho_{it})R_{it} = \hat{\delta}x_{it} + \hat{u}_{it} \\ ratiolr &:= (1+\rho_{it})r_{it} = \hat{\delta}x_{it} + \hat{u}_{it} \end{aligned} \tag{7}$$

where i from 1 to 114 is the identification of the country and t that of the year (1967-2021). The estimated unitary pure interest is $\hat{\delta}$ and $\hat{u}_{it}$ the residual. Taking into account the stability on time of the proportion of interests according to López-Laborda and Peña[2] (2018), we hope the independent variable will also be stable, so we consider a "fix" variable made by ones called *one*.

The second set of empirical specifications is given by transforming expression (1) into a model adapted to OLS methodology for financial services. The econometric specification for RL and RL2 is:

$$\begin{aligned} ratioln(RL) &:= \ln\left(\frac{IP}{IR}\right) = \frac{1}{\hat{\alpha}}x_{it} + \hat{u}_{it} \\ lnlddcn(RL2) &:= \ln\left(\frac{IR\ S}{IP\ C}\right) = \hat{\beta}x_{it} + \hat{u}_{it} \end{aligned} \tag{8}$$

where the deposits would be S and the credit C as the financial stocks instead of population, being $\hat{\alpha}$

the reciprocal of the coefficient of the independent variable of RL and $\hat{\beta}$ the coefficient of the independent variables of RL2 by considering $\alpha = 1$ and the same distance. So, the four specifications are provided.

The variables are taken from the World Bank[15] (2024) Database. The panel consists of 114 countries between the years 1967 and 2021. The unbalanced panel used has been the full dataset (all the 216 countries from 1960 to 2021[1]) from the World Bank for those variables. The summary of the main statistics of the variables is collected on **Table 1**.

Table 1. Descriptive statistics of the variables.

Variable	Observations	Mean	Std. Dev.	Min	Max
deprate	4599	44.752	1935.350	-0.419	130592.000
lendrate	4473	39.404	1493.189	0.5	99764.530
dep	8181	2.67E+13	4.48E+14	-9.81E+13	2.85E+16
cred	8214	3.70E+13	3.76E+14	-5.42E+13	1.02E+16
ratiodr	3953	0.316	0.124	-0.188	0.932
ratiolr	3953	0.684	0.124	0.068	1.188
rho	3953	0.368	0.248	-0.865	1.377
delta	3953	0.401	0.109	-0.448	0.5
deldr	3953	0.401	0.109	-0.448	0.5
dellr	3953	0.401	0.109	-0.448	0.5
one	13392	1	0	1	1
ratioln	3946	-0.881	0.811	-6.215	2.622
lnlddcn	2794	3.345	2.148	-5.280	16.158

The first two variables to highlight are the deposit and lending interest rates, respectively, *deprate* and *lendrate*. The variable *dep* are the gross savings amounts, *cred* the net credit amounts, while the shares of *deprate* and *lendrate* over the sum of both of them are, respectively, *ratiodr* and *ratiolr*. The mobile ratio ρ of the interest rates is *rho*, *delta* is the pure interest and the unitary pure interest, both of them according to (6), *one* is the fix variable of value 1, and the *deldr*, *dellr* and *ratioln*, *lnlddcn* are defined in, respectively, the above and below equations of (7), (8).

For checking the main assumptions for OLS models: normality of the error terms, absence of heteroscedasticity and autocorrelation problems among error terms, and absence of multicollinearity among independent variables, the next strategy has been followed.

First, the Jarque-Bera (J-B), Skewness and Kurtosis test have been analyzed country by country. Second, two kinds of models with OLS have been applied: first, with standard errors robust to heteroscedasticity, and

[1] Even dealing with panel data, OLS methods have been used since our interest is to analyze the dependent variables in a country-by-country way, that is the reason for using this cross-section strategy even for panel data.

second, robust to both heteroscedasticity and autocorrelation by using clustered standard errors. Finally, in the next models multicollinearity will not be necessary to test since only one independent variable will be applied. Next **Table 2** shows the p-values of the different normality tests for the residuals of the models of the next **Tables 3-4**, that is, the empirical evidence.

Table 2. Normality tests with the residuals of the models of **Tables 3** and **4**, by OLS.

Dependent variable	***deldr***	***dellr***	***ratioln***	***lnlddcn***
Independent variable: One				
J-B test Average p-value	0.3221	0.3221	0.3884	0.5167
Skewness test Average p-value	0.3123	0.3123	0.3635	0.4368
Kurtosis test Average p-value	0.3378	0.3378	0.3850	0.5346

The results show in all cases the non-rejection on average of the null hypothesis of normality of the model residuals.

5.2. Empirical evidence

The results for the four specifications collected in the above and below equations of (7) and (8) appear in, respectively, from the second to the fifth column of **Table 3** for robust standard errors and **Table 4** for clustered standard errors controlling for possible heteroscedasticity and autocorrelation.

Table 3. OLS methodology applied to the interaction models with robust standard errors.

Dependent variable	***deldr***	***dellr***	***ratioln***	***lnlddcn***
Independent variable: One	0.401***	0.401***	-0.881***	3.345***
	(0.002)	(0.002)	(0.013)	(0.041)
Obersvations (N)	3953	3953	3946	2794
Adjusted R^2	0.932	0.932	0.541	0.708

Note: *p-value(p)<0.1, **p<0.05, ***p<0.01, Standard Errors in parenthesis, below the coefficient.

Table 4. OLS methodology applied to the interaction models with clustered standard errors.

Dependent variable	***deldr***	***dellr***	***ratioln***	***lnlddcn***
Independent variable: One	0.404***	0.404***	-0.861***	3.268***
	(0.003)	(0.003)	(0.013)	(0.052)
Obersvations (N)	2000	2000	1994	1606
Adjusted R^2	0.928	0.928	0.535	0.708

Note: *p-value(p)<0.1, **p<0.05, ***p<0.01, Standard Errors in parenthesis, below the coefficient.

The results show a similar explanatory power, coefficient and its significance for both FG specifications, while there is a higher explanatory power for RL2 than for RL The same estimations are performed but considering only each one of the years between 1967 and 2021 in the next empirical illustrations, collecting their coefficients in **Figure 1**, their R^2 and that of their differences in **Figure 2**, respectively, above and below. Only one of the FG specifications have been used in the estimations, since their results are the same. These results that show the huger superiority of FG models on explanatory power measured by R^2, and they also show the stability of the coefficient of the last models with a value around the average of 0.4.

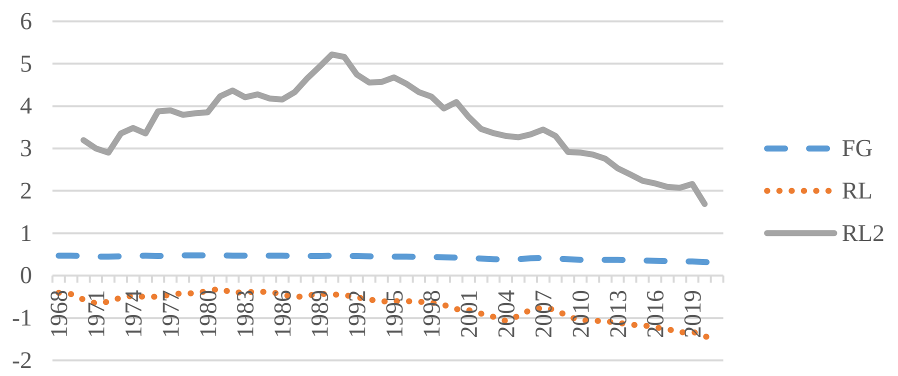


Figure 1. Coefficient of the annual specifications of Gravity Models for financial services.

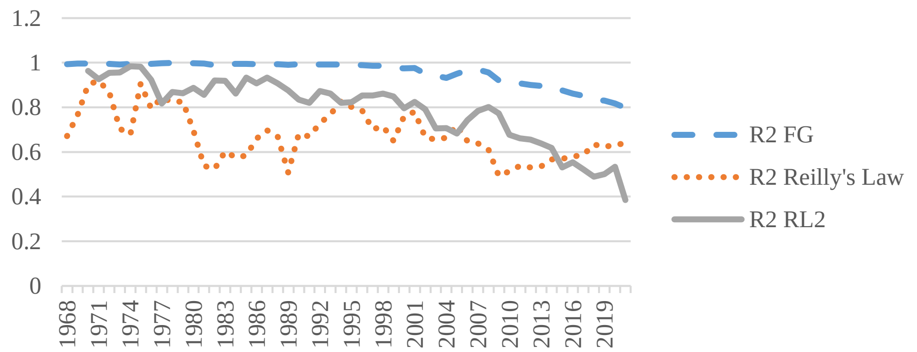


Figure 2. R^2 of the three specifications of Gravity Models and their differences.

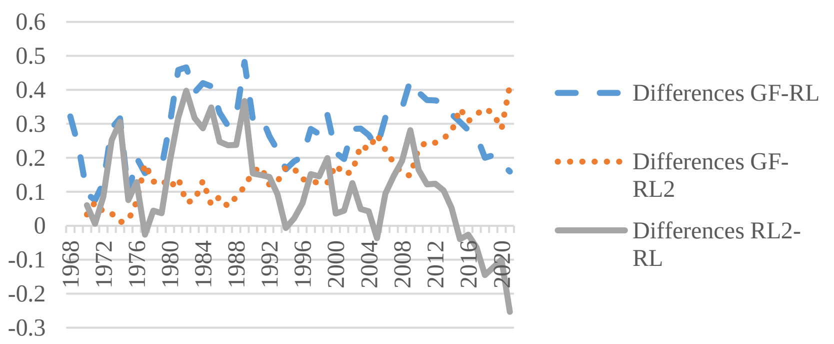
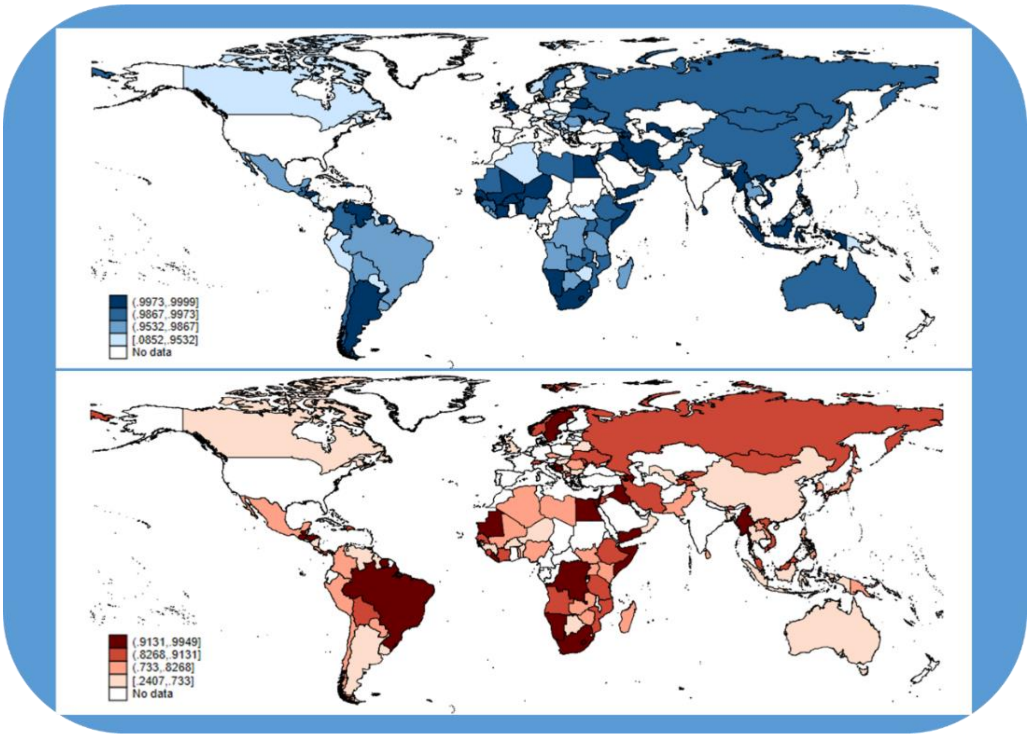
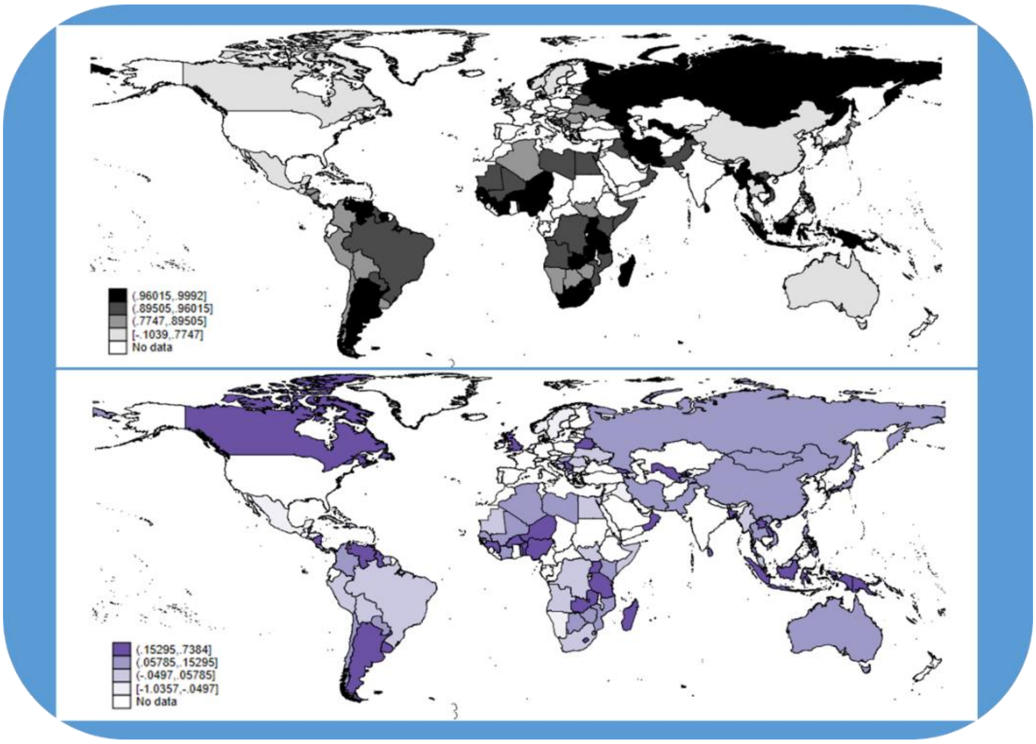


Figure 3. R^2 of the differences between the three specifications of Gravity Models.

Maps 1-3 show the R^2 of, respectively, the FG(GF)/RL models, RL2/Differences between RL2 and RL models and the differences between FG-RL/FG-RL2 (above/below).

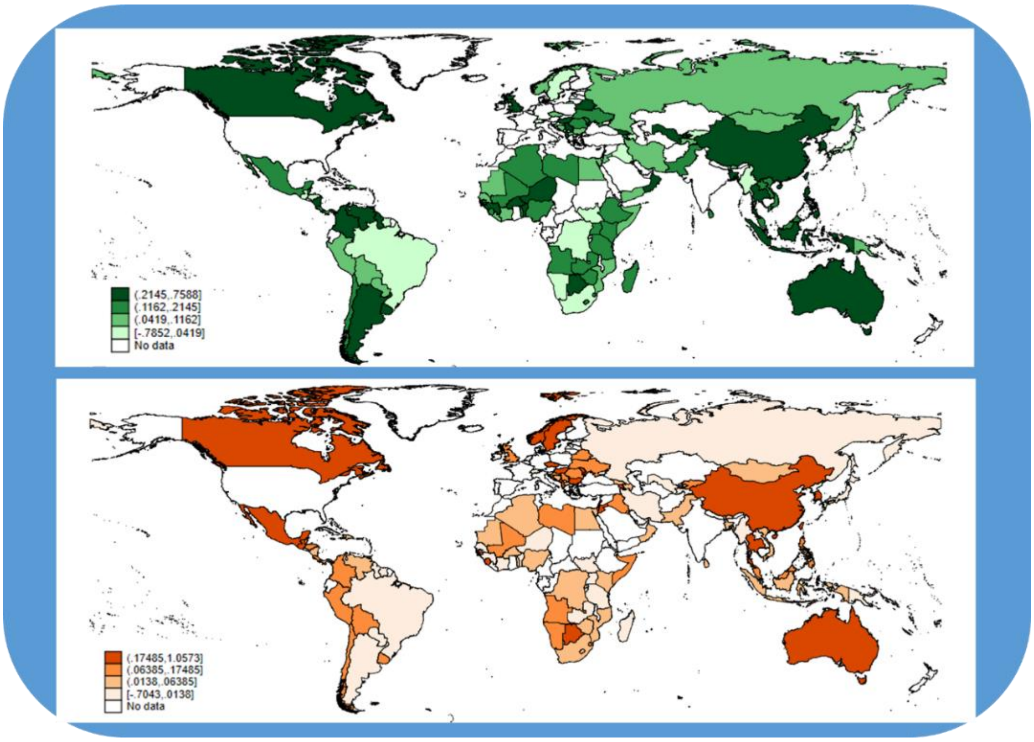


Map 1. R^2 of the FG/RL models (above/below).



Map 2. R^2 of the RL2/ RL2-RL models (above/below).

The maps show a higher difference in the R^2 between FG and RL/RL2 models in big countries as Canada, China or Australia, while it is high for the three specifications in big countries as Brazil or Russia. Furthermore, developed countries as UK or Sweden benefit from a higher R^2 in the FG models than other countries.



Map 3. R^2 of FG-RL/FG-RL2 models (above/below).

Finally, there is a t-test of averages for checking the equivalence of the two specifications of FG models between them and respect to the other variable they are equal to according to the proposed theory: the unitary pure interest delta. With the differences between each couple of variables equal to zero as null hypothesis and otherwise as alternative, the p-values of the results are shown in **Table 5**.

Table 5. P-value of the t-test for the differences between deltadr, deltalr and delta.

diff	Ha: mean(diff) < 0	Ha:mean(diff) ≠ 0	Ha: mean(diff) > 0
deldr-delta	0.2345	0.4691	0.7655
deldr-dellr	0.5517	0.8967	0.4483
dellr-delta	0.2144	0.4287	0.7856

The results show that the three expressions are statistically equivalent on average, confirming the suitability of the FG models for financial services.

6. Discussion and concluding remarks

The main target of this paper, the adaptation of the gravity models for financial services in an operational and empirical way has been fulfilled. Based on previous Financial Gravity (FG) models and after some modifications, two interactional models explaining the unitary pure interest have been developed. These models have been compared with a new proposal of two adaptations of the well-known Reilly's Law model of regional sciences to financial services. Handling with a non-balanced panel dataset of 114 countries from 1967 to 2021 and applying OLS econometric techniques, the results show several relevant information.

So, the main empirical findings are five. First, the most explanatory models are the adapted FG models, with extremely high R^2 higher than 0.9, in contrast to the Reilly's Law adaptations with values around 0.7. Second, the "stability" of the share of value added of financial services, and so, of the unitary pure interest, is empirically checked, observing an estimated coefficient representing that variable with a zero annual trend. Third, the explanatory power of the FG models have stayed stable close to 1 until the beginning of the 2000s, close to the dot-com crisis, when it slightly decreases, with a little bit more pronounced download after the end of the Great Recession, around 2009, reducing its difference with the R^2 of the other models. Fourth, the big countries benefit from a higher value of the R^2 of gravity models rather than the smaller ones, with a more pronounced explanatory power for these big countries, and also for developed countries, with the application of FG models rather the other specifications. Finally, t-test have been developed showing the equivalence of the equalities proposed in the adaptations of the FG models.

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Conflict of interest

The author declares no conflict of interests.

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