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RESEARCH ARTICLE

Capital gains tax versus wealth tax

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ABSTRACT

This article provides a financial analysis of capital gains tax and wealth tax and their impact on investors' decision-making behaviour and corporate finance. First, the capital gains tax is analysed according to current German tax law, revealing a negative impact on firms and the economy. This finding is particularly worrying in an economic crisis, and it highlights the need to consider alternative tax options. A detailed analysis of wealth taxes follows, which shows that a special wealth tax on safe assets represents a promising alternative to current tax practice in view of the ongoing socio-ecological transformation.

Keywords: finance; capital gains tax; wealth tax; tax law; economic crisis; negative interest rates

1. Introduction

Currently, many economies around the world are experiencing persistent stagnation. Since taxes play a central role in investment and economic recovery, it is crucial to consider how capital assets are taxed and its effects on the decision-making behaviour of investors and corporate finance.

This article first analyses the capital gains tax and its impact on investors' return expectations from a classical finance perspective. The analysis shows that this type of taxation has a negative effect on corporate finance and the economy. Two alternatives to taxing capital assets are then examined: first, interest income tax as a possible variant of capital gains tax and, second, a general and a special wealth tax. The advantages and disadvantages of each type of tax are discussed in detail before the article concludes by finding decisive arguments in favour of a special wealth tax on safe assets.

2. Classical finance

2.1. The standard model

The standard model of the capital market was developed by William Sharpe in the 1960s[1], among others, leading to his being awarded the Nobel Prize in Economics in 1990. The model provides the standard return equation for pricing real assets (common stocks, shares, real estate, private equity, etc.). It indicates what return (r_j) an investor can expect ex ante from a certain real asset j at a future point in time:

$$E(r_j) = r_f + E(RP_j) \quad \text{for} \quad r_f > 0, \quad E(RP_j) > 0 \quad \text{and} \quad j = 1, 2, 3, \ldots n \tag{1}$$

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In Eq. (1), the expected return $E(r_j)$ of a single real asset is the sum of the risk-free interest rate (r_f) and a risk premium (RP_j) that covers the market risk of the asset. In the European Union (EU), the risk-free interest rate is represented by the Euribor rate. The risk premium depends on the individual firm and the respective industry and varies between approximately 2%–5% per Annum (p.a.). The expected value operator is denoted by $E(\cdot)$.

The return equation (1) is also relevant for corporate finance. Given the following identity,

$$\text{cost of capital for equity} \equiv \text{return expectations of investors}, \tag{2}$$

firms calculate the cost of capital for equity according to the standard return equation (1), which requires firms to generate the return expected by investors through capital costs[2].

2.2. Limits of the standard model

The standard model, as represented in Eq. (1), is based on a perfect capital market and therefore on strongly idealized assumptions, excluding taxes, subsidies and other market imperfections. This has the advantage of allowing the model to have a simple and clear structure, but there are also theoretical drawbacks. In the standard model (1), both the risk-free interest rate and the risk premium of real assets are positive, so that investors always expect a positive return overall, usually 5% p.a. or more. Consequently, investments are only made in those firms and industries promising high returns.

In an economic crisis, corporate performance is weakened, and firms may have difficulties meeting the return expectations of their investors. To prevent investors from exiting the market, firms could generate expected returns by taking evasive measures that harm the common good, such as externalizing environmental and social costs. Furthermore, companies may lack the financial resources to develop and implement innovative policies, such as CSR, which would exacerbate their economic and societal situation and endanger sustainable economic development.

2.3. Considering taxes

Two aspects are important when making investment decisions about real assets. First, investors always base their decisions on a risk-free reference interest rate in order to calculate a risk premium that covers the market risk of the investment[3,4]. Second, taxes imposed on capital assets must also be taken into account:

> In general, … investors will demand after-tax efficient portfolios, and those portfolios may not be before-tax efficient. Long, 1977, p. 29[5]

In this scenario, the original standard model (1) is no longer applicable. When taxes are included in the standard model, they become additional parameters that need to be considered[6,7]. Extensions of the standard model assume that the market risk of real assets is fully covered by an adequate risk premium, even when taxes are taken into account. Tariff and tax collection processes also play an important role in modelling taxes. In Germany a proportional tariff (flat rate) applies, and the tax is automatically paid as a withholding tax by the account-holding financial institution (bank, etc.).

3. Capital gains tax

3.1. Withholding tax

A capital gains tax (interest, dividends and other capital gains) has been levied in Germany since 2009. It offers several advantages. It provides a uniform, proportional tax rate of 25% (flat rate) for natural and legal persons as well as a withholding tax deduction, which ensures a uniform and unbureaucratic collection based on the Austrian model. Additionally, a tax allowance of €1,000 currently protects small savers from

excessive taxation. The flat rate also enables for all taxable investors the uniform calculation of interest and returns after taxes and enables firms to more easily calculate the cost of capital.

3.2. Modelling a capital gains tax

The starting point of modelling capital gains taxation is calculating the risk-free interest rate after taxes ($r_{f,at}$). Based on a capital gains tax (τ), this value is calculated according to the well-known formula:

$$r_{f,at} = (1 - \tau) \cdot r_f \qquad \text{for} \quad r_f > 0 \quad \text{and} \quad \tau \in (0,\ 1) \tag{3}$$

Eq. (3) applies to the amounts invested about the tax allowance.

How does a capital gains tax affect investment decisions in real assets? Eq. (3) defines a new, risk-free reference interest rate for taxable investors, which they then use to calculate an appropriate risk premium. If investors can realize a risk premium that fully covers the market risk of the investment, real assets are generally attractive to them. Under this premise, investors calculate ex ante the following return equation after taxes:

$$E(r_j)_{at} = [\ r_{f,at} + E(RP_j)\ ] \ / \ (1 - \tau) = r_f + E(RP_j) \ / \ (1 - \tau) \tag{4}$$

$E(r_j)_{at}$ is the expected return when investors consider ex ante all taxes that arise at the shareholder level when making investment decisions (see Appendix). Since a capital gains tax is a withholding tax with a flat rate, the return equation after taxes (4) applies equally to all taxable investors.

Eq. (4) states that taxable investors base their investment decisions on the taxed interest rate ($r_{f,at}$) and calculate ex ante – in line with the standard model (1) – a risk premium (RP_j) that covers the market risk of the investment. The term $(1 - \tau)$ appears in the denominator of Eq. (4) in order to offset ex ante for the future deduction through capital gains tax. This is intended to ensure that the calculated risk premium can be fully realized.

Example 1 below shows that, due to the capital gains tax, a taxable investor calculates a higher return of 6% for a real asset. In the standard model (1), however, the expected return would be only 5% (2% + 3%). Due to the identity (2), the capital gains tax also increases the cost of capital for firms by 1% compared to the standard model (1).

Example 1

Assume a risk-free interest rate of 2% (Euribor) and a capital gains tax of 25% p.a. For a risk-free bank account, a taxable investor receives an after-tax interest rate according to Eq. (3):

$$(1 - 0.25) \cdot 2\% = 1.5\%$$

In the case of a real asset, an investor adds a risk premium that covers the market risk of the investment, e.g. 3% p.a. and demands in sum a return of

$$1.5\% + 3\% = 4.5\%$$

This is the rate of return that the investor ultimately demands in advance (ex ante), even if he has to pay capital gains tax at the end of the year (see Appendix). In order to realize this return without reduction, he must calculate a higher return at the beginning of the year according to Eq. (4):

$$E(r_j)_{at} = [\ 1.5\% + 3\%\ ] \ / \ (1 - 0.25) = 6\%$$

Let us assume that the investor **actually** achieves (ex post) a return of 6% at the end of the year and pays capital gains tax of 25%, then we get:

$$(1 - 0.25) \cdot 6\% = 4.5\%$$

In this way, the investor can ultimately realize a return that fully includes the initially calculated risk premium of 3%.

3.3. Disadvantages of a capital gains tax

How does a capital gains tax affect investors' investment behaviour? The after-tax return equation (4) implies that rational investors do not willingly forgo a risk premium that fully covers the market risk of an investment. Therefore, they calculate ex ante a higher return, as expressed in Eq. (4) by the divisor $(1 - \tau)$.

How does a capital gains tax affect corporate finance? The after-tax return equation (4) states that a capital gains tax increases investors' return expectations (Example 1). Due to identity (2), this increases the cost of capital, making corporate financing more expensive. Investors expect higher returns as a result of the tax (τ), which then have to be generated by firms in the form of higher capital costs. Although the tax (τ) is applied at the shareholder level, by making equity financing more expensive it ultimately has an effect at the corporate level.

3.4. The case for a capital gains tax

The aim of a capital gains tax is to ensure that capital income is taxed equally, but it fails to account for the significant differences in risk between safe assets and real assets. Real assets are subject to double taxation, once at the corporate level through corporate tax and again at the shareholder level through capital gains tax [8, 9]. Double taxation could be avoided if safe assets were taxed and real assets were exempted. This could be achieved with an interest income tax on safe assets.

3.5. Interest income tax

One variant of the capital gains tax is the interest income tax, which only applies to safe assets such as savings deposits, overnight and fixed-term deposits, money market funds and government bonds. This type of tax avoids the disadvantages of capital gains tax mentioned in Sections 3.3 and 3.4. Analogously to Eq. (3), the risk-free interest rate after taxes ($r_{f,at}$) is calculated using an interest income tax (τ_f):

$$r_{f,at} = (1 - \tau_f) \cdot r_f \qquad \text{for} \quad r_f > 0 \quad \text{and} \quad \tau_f \in (0,\ 1) \tag{5}$$

The after-tax return equation is:

$$E(r_j)_{at} = r_{f,at} + E(RP_j) = (1 - \tau_f) \cdot r_f + E(RP_j) \tag{6}$$

Eq. (6) states that the risk-free interest rate after taxes ($r_{f,at}$) is the new risk-free reference interest rate for the return expectations of taxable investors.

How does an interest income tax affect investors' return expectations and corporate finance? In contrast to the standard model (1), the risk-free interest rate (r_f) in Eq. (6) is reduced after multiplication by the factor $(1 - \tau_f)$. This also reduces investors' return expectations, represented by the value $E(r_j)_{at}$ on the left-hand side of Eq. (6). Given identity (2), an interest income tax ultimately also reduces the firms' capital costs.

3.6. The case for an interest income tax

Does an interest income tax ensure an equal taxation of capital income? At first sight this does not seem to be the case, as this tax only applies to safe assets. On closer inspection, however, it turns out that an interest income tax ultimately causes the same tax reduction of income for real assets as it does for safe assets:

$$\text{Tax-related reduction in income} = \tau_f \cdot r_f \tag{7}$$

According to Eq. (7), there is an **actual** reduction in income for safe assets (Eq. 5) and, even if real assets are not actually taxed, an ex ante **imputed** reduction in income calculated for real assets (Eq. 6). This phenomenon provides risk-adjusted equal treatment of both safe and real assets.

3.7. Discussion

Based on Eq. (6), an interest income tax eases financing conditions for firms, as investors reduce their return expectations, which in turn lowers the firms' cost of equity. However, this effect is limited. With a tax rate of 100%, the risk-free interest rate after taxes ($r_{f,at}$) would be zero and Eq. (6) would give:

$$E(r_j)_{at} = 0 + E(RP_j) = E(RP_j) \tag{8}$$

Equation (8) states that even with a maximum interest income tax rate of 100%, firms must generate an appropriate risk premium of between 2%–5%, depending on the firm and industry.

However, in a recession, it is difficult for firms to satisfy even the reduced Eq. (8). In such a scenario, only a negative interest rate after taxes ($r_{f,at} < 0$) would ensure that real assets remain attractive for investors. If the earnings situation of firms is poor, then only negative interest rates after taxes would allow firms to offer their investors a positive risk premium. But this cannot be achieved with an interest income tax, because this tax cannot tax interest income at a rate higher than 100%. A special wealth tax (discussed in Section 4.4) would provide a solution to this dilemma.

4. Wealth tax

4.1. Tax law

Wealth taxes were abolished in Germany in 1997 following a decision by the Federal Constitutional Court that they violated the principle of equality[10]. The court set two conditions for their use: first, that a wealth tax must ensure uniform taxation of different types of assets and, second, that it must be calculated in such a way that it can be paid from the expected capital income. A tax model that does not meet these conditions requires a special justification.

4.2. General wealth tax

A general wealth tax affects the total assets (including equities, bonds, cash, real estate, etc.) of natural and legal persons[11]. What effect does this tax have on the decision-making behaviour of investors?

For a wealth tax (υ), the risk-free interest rate after taxes ($r_{f,at}$) is calculated with the following formula (Fahrbach, 2014, p. 95):

$$r_{f,at} = (1 + r_f) \cdot (1 - \upsilon)-1 \approx r_f - \upsilon \qquad \text{for} \quad r_f > 0 \quad \text{and} \quad \upsilon \in (0,\ 1) \tag{9}$$

Eq. (9) applies to the amounts invested above the tax allowance.

Eq. (9) provides the starting point for calculating the after-tax return equation for real assets:

$$E(r_j)_{at} \approx [\ r_{f,at} + E(RP_j)\ ] + \upsilon \tag{10}$$

Eq. (10) states that taxable investors base their investment decisions on the risk-free interest rate after taxes ($r_{f,at}$) and add a risk premium (RP_j) that covers the risk of the investment. In addition, investors add the wealth tax rate (υ) in Eq. (10) in order to anticipate a reduction in returns due to the wealth tax at the end of the year (see Appendix). This ensures that investors can fully realize the calculated risk premium (RP_j). From Eq. (9) and (10) we finally arrive at:

$$E(r_j)_{at} \approx [\ r_f - \upsilon + E(RP_j)\ ] + \upsilon = r_f + E(RP_j) \tag{10}$$

This result is surprising because, although Eq. (10) models the return expectations of investors after taxes, no taxes are included in this equation at all. It is the standard model (1) without taxes. In other words, a general wealth tax has no effect on the return expectations of investors.

4.3. Discussion

At first glance, since a general wealth tax imposes additional taxes on firms at the shareholder level, one might think that it has the same disadvantages as a capital gains tax. However, the analysis in Section 4.2 shows that this is not the case: A general wealth tax has no effect at all on investors' return expectations or on corporate finance. The return equation after taxes (10) fully corresponds to the standard model (1), which excludes taxes. In this sense, a general wealth tax, unlike a capital gains tax, enables a neutral taxation of real assets[12]. However, a general wealth tax has the same disadvantages as the standard model (1), which were outlined in Section 2.2: namely, that in a recession, firms have difficulty generating profits that meet investors' demands. This disadvantage could be offset by a special wealth tax on safe assets, as discussed below in Section 4.4.

Mathematically, a capital gains tax has no advantages over a wealth tax, because a capital gains tax (τ) can always be converted into a wealth tax (υ), but not vice versa. From Eq. (3) and (9) we obtain approximately:

$$\upsilon \approx r_f \cdot \tau \qquad \text{for} \quad r_f > 0 \quad \text{and} \quad \tau, \upsilon \in (0, 1) \tag{11}$$

Eq. (11) can be illustrated with a simple example: With an interest rate of 2%, it is irrelevant for an investor whether he or she pays a 25% capital gains tax or a 0.5% wealth tax, as ultimately the same tax has to be paid. Otherwise, a wealth tax cannot always be converted into a capital gains tax, because the latter cannot be higher than 100%. Mathematically, concerns about the viability of a wealth tax are therefore unfounded.

4.4. Special wealth tax on safe assets

To overcome the disadvantages of a general wealth tax, one could define a special wealth tax that only applies to safe assets and exempts real assets. A special wealth tax would apply to the substance of safe assets and therefore also to non-interest bearing bank accounts (current accounts). This tax would also allow for higher taxation than a capital gains tax. Moreover, as a special wealth tax could replace the existing capital gains tax, no additional tax would be necessary.

4.5. Modelling of a special wealth tax

In the case of a special wealth tax on safe assets (υ_f), the risk-free interest rate after taxes ($r_{f,at}$) can be calculated analogously to Eq. (9):

$$r_{f,at} \approx r_f - \upsilon_f \qquad \text{for} \quad r_f > 0 \quad \text{and} \quad \upsilon_f \in (0, 1) \tag{12}$$

Eq. (12) applies for amounts invested above the tax-free allowance. For real assets, the return equation after taxes is approximately [13, 14]:

$$E(r_j)_{at} = r_{f,at} + E(RP_j) \approx r_f - \upsilon_f + E(RP_j) \tag{13}$$

Eq. (13) states that the risk-free interest rate after taxes ($r_{f,at}$) is the new reference interest rate for taxable investors. They would base their investment decisions on this value ($r_{f,at}$), which they then use to calculate an appropriate risk premium (RP_j) that covers the market risk of the asset. Compared with the standard model (1), the tax rate (υ_f) remains as an additional parameter in Eq. (13). If the wealth tax (υ_f) is a withholding tax with a flat rate, then the return equation after taxes (13) is equally valid for all taxable investors.

How does a special wealth tax (υ_f) affect investors' return expectations and corporate finance? In Eq. (12), the risk-free interest rate (r_f) is reduced by a special wealth tax (υ_f). Consequently, the expected return on a real asset, on the left-hand side of Eq. (13), is also reduced. This results in lower return expectations for investors on real assets and, due to identity (2), also lower capital costs for firms.

4.6. Negative interest rates after taxes

A notable feature of the special wealth tax is that the risk-free interest rate after taxes can be negative whenever, according to Eq. (12), the wealth tax rate (υ_f) exceeds the risk-free interest rate:

$$r_{f,at} < 0 \quad \text{for} \quad \upsilon_f > r_f$$

Example 2 illustrates the impact of negative after-tax interest rates on investors' return expectations. When the risk-free rate after taxes ($r_{f,at}$) is negative, taxable investors accept a lower return on real assets, in this example 1%, which allows firms to operate at low profitability. However, negative interest rates after taxes would require that banknotes (paper money) be eliminated and payments largely digitized in order to prevent the flight to cash[15].

Example 2

Assume a risk-free interest rate of 2% (Euribor) and a special wealth tax on safe assets of 4% p.a. For a risk-free bank account, a taxable investor receives a negative after-tax interest rate according to Eq. (12):

$$r_{f,at} \approx 2\% - 4\% = -2\%$$

In the case of a real asset, an investor adds a risk premium that covers the market risk of the investment, e.g. 3% p.a. and then calculates the expected return according to Eq. (13):

$$E(r_j)_{at} \approx -2\% + 3\% = 1\%$$

This result shows that the investor can realize the initially calculated risk premium of 3% even if he expects a very low return of 1%.

4.7. Steering function

A special wealth tax on safe assets can be given a steering function[16]. If safe assets are taxed more heavily, there will be greater demand for real assets. In addition, as taxable investors then revise their return expectations downward, they would be willing to provide firms with cheap equity capital. Especially in an economic crisis, the government could shift the after-tax interest rates into the negative territory to provide further relief for firms and the economy. Depending on the economic situation, a uniform withholding tax with a flat rate of 3%–5% p.a. at EU level would be preferable to the different national capital gains taxes of the EU member states. This would allow the EU to ensure a favourable investment climate even in times of positive interest rates and a weak economy.

4.8. The case for a special wealth tax

The legal requirements for a wealth tax were specified in Section 4.1[10]: different assets must be taxed uniformly, and the substance of the assets should not be reduced. Does a special wealth tax on safe assets meet these requirements? One could object, first, that safe assets are taxed and real assets are not and, second, that a special wealth tax could lead to negative after-tax interest, which would reduce the substance of the assets.

The first objection can be met, analogously to interest income tax in Section 3.6, by the principle of risk-adjusted equal treatment, because a special wealth tax causes the same tax-related reduction in income for both safe and real assets:

$$\text{Tax-related reduction in income} \approx \upsilon_f \quad (14)$$

The reduction in income, according to Eq. (14), occurs **de facto** in the case of safe assets (Eq. 12) and only **calculatory** in the case of real assets (Eq. 13), since the latter are not actually taxed. A special wealth tax thus ensures, when adjusted for risk, equal tax treatment of both safe and real assets.

The second objection concerns the substance of the assets. With a special wealth tax, after-tax interest can become negative, which would in fact reduce the substance. A special justification is needed. The steering function that can be given to a special wealth tax (discussed in Section 4.7) fulfils that requirement. Economic crises can be countered by negative interest rates after taxes to ensure favourable financing conditions for firms to realize all planned investments.

5. Conclusion

This article first analyses the capital gains tax under current German tax law and its impact on investor decision-making and corporate finance. The analysis shows that the capital gains tax has obvious disadvantages as investors expect higher returns, which increases the cost of equity capital and thus for corporate finance. One possible solution would be to exempt real assets from taxation at the shareholder level.

Alternatives to a capital gains tax are then considered: namely, an interest income tax, a general wealth tax and a special wealth tax. The analysis only applies to those amounts that are invested above the tax allowance. A welcome simplification is that these taxes can be modelled as a flat-rate withholding tax, making it possible to directly compare the advantages and disadvantages of the different types of tax.

This article is also about the role of tax policy in an economic crisis. Even if firms have difficulties to operate profitably, they must still offer investors a sufficiently high risk premium. Investors have justifiable expectations for an appropriate risk premium when holding real assets. The government should therefore use tax policy measures to ensure that investors are offered this risk premium even in an economic crisis.

These considerations ultimately lead us to a special wealth tax on safe assets (savings deposits, overnight and fixed-term deposits, money market funds and government bonds, etc.), but exempts real assets (common stocks, shares, real estate, private equity, etc.). The analysis of this type of tax leads to a new tax law principle that provides for risk-adjusted equal treatment resulting in the same (calculated) reduction in after-tax income for both safe and real assets.

The main advantage of a special wealth tax is that it can be given a steering function. This is particularly important in an economic crisis, because it enables the government to shift after-tax interest rates into negative territory. With negative interest rates after taxes investors would revise their return expectations downwards and be willing to provide cheap equity to firms, which would enable them to operate at low profitability. Favourable financing conditions enables firms to make much-needed investments in sustainability transformation even in a weak economic environment.

Conflict of interest

The authors declare no conflict of interest.

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Mathematical Appendix

Assumption 1: Capital assets (risk-free and risky assets) are taxed annually at the shareholder level.

Assumption 2: For a risk-averse investor, realizing an adequate risk premium for a real (risky) asset takes priority over paying taxes, i.e. an investor expects a definite risk premium, even if he has to pay taxes, otherwise he will prefer a risk-free bank account.

Assumption 3: For the sake of simplicity, it is assumed that the risk premium depends solely on market risks and is mathematically independent of taxation.

Theorem 1: Under Assumptions 1-3, the term $r_{f,at} + E(RP_j)$ denotes the rate of return on a real asset that an investor demands in advance (ex ante) but **after** deducting taxes that are due at the shareholder level at the end of a year. This implies that taxable investors (a) base their expectations for real assets on the risk-free interest rate after taxes ($r_{f,at}$) and (b) expect a definite risk premium (RP_j), even after paying taxes at the end of the year.

Proof: The Theorem 1 follows directly from the Assumptions 1-3.

Definition 2: Under Assumptions 1-3, the term $E(r_j)_{at}$ denotes the expected rate of return if an investor considers ex ante all relevant taxes, which apply at the shareholder level.

Assumption 4: Capital assets (risk-free and risky assets) are taxed with an annual capital gains tax (τ).

Theorem 2: Under Assumptions 1-4, $E(r_j)_{at} = [\, r_{f,at} + E(RP_j) \,] / (1 - \tau)$.

Proof: A taxable investor demands a rate of return according to Theorem 1. But in order to achieve this return, he calculates ex ante a higher return according to Theorem 2. Let us assume that at the end of the year the investor **actually** achieves an ex post return according to Theorem 2, then he has to pay capital gains tax, which reduces the return to

$$(1 - \tau) \cdot E(r_j)_{at} = (1 - \tau) \cdot [\, r_{f,at} + E(RP_j) \,] / (1 - \tau) = r_{f,at} + E(RP_j)$$

The result is the original term according to Theorem 1. QED

Assumption 5: Capital assets (risk-free and risky assets) are taxed with an annual general wealth tax (υ).

Theorem 3: Under Assumptions 1-3 and 5, $E(r_j)_{at} = r_f + E(RP_j)$.

Proof: A taxable investor demands a rate of return according to Theorem 1, which is in the case of a wealth tax approximately:

$$r_f - \upsilon + E(RP_j)$$

But in order to achieve this return, he calculates ex ante a higher return according to Theorem 3. Let us assume that at the end of the year the investor **actually** achieves an ex post return according to Theorem 3, then he has to pay a wealth tax, which reduces the return approximately to

$$E(r_j)_{at} - \upsilon = [\, r_f + E(RP_j) \,] - \upsilon = r_f - \upsilon + E(RP_j) \approx r_{f,at} + E(RP_j)$$

Since the tax rates (υ) cancel each other out in this proof, the result corresponds not only approximately but exactly to the Theorem 1. QED