Original Research Article

Production policy based on echelon base stocks for reverse logistics and industrial symbiosis: an IPA approach

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Abstract: Last years, the implementation of reverse loops in logistics and the deployment of industrial symbiosis becomes more important. However, these new activities management isn't easy and it's necessary to propose methodologies to facilitate the actors work. However, the existing studies are more at strategic level, aiming with implementation or cooperation. At the tactical/operational level, solutions are poorly effective and/or expensive. Our work targets an optimal production policy definition based on the base stock strategy adapted for an industrial symbiosis. The system is composed of two kinds of warehouses and three types of enterprises. The recovering depends on the kind of collected used finished products. All demands are uncertain such as the number of collected used finished products and generated waste. Then, the main objective is to minimize the sum of all economic, environmental and social costs by identifying the level of the base stocks. To do this, an infinitesimal perturbation analysis study is conducted to evaluate the gradient estimators of the objective function subject to an echelon base stock production policy. This result is then used in a simulation based-optimization algorithm to determine these stock levels and highlight our theoretical results by comparing with other replenishment strategy and mathematical programming.

Keywords: Inventory control, production planning and scheduling, industrial symbiosis, echelon base stock policy, stochastic fluid model, infinitesimal perturbation analysis.

1. Introduction

Since the publication of the last IPCC reports (see http://ipcc.ch), it appears crucial to change the paradigm and propose new solutions within a sustainable development framework to limit our impact on the environment but also on society[1]. To achieve this, an idea is to adopt strategies inspired by nature[2]. The principle is to emulates nature's designs, processes and organization, and reconnect anthropic activities to define sustainable solutions[3]. Many strategies have been proposed[4], and among all the done works, a particular interest lies in logistic flows with two directions. The first one concerns the incorporation of environmental (and sometimes) social aspects. In this case, authors talk about sustainable or green systems and they develop strategies for respectful sourcing and distribution of materials[5]. The second direction concerns the integration of end-of-life products. In this last case, authors talk about circular economy with mainly reverse logistics[6] and reusing/recovering activities[7]. For this present work, we consider the symbiotic and reverse flows between actors. The benefits of reverse logistics are well documented[8] but they are difficult to implement and manage because of collection and recovery activities, reuse without any modification or remanufacturing due to uncertainties (quality and quantity or collected products, time of the return for these products[7,9]). For symbiotic flows, the advantages are more related to society even if, economic and environmental impacts also appear[10].

The principle of symbiotic flows comes from the definition of biological symbiosis, which namely refers to a close, long-term interaction between different species. The concept of industrial symbiosis (IS) is

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defined[11] as an industrial system that "in addition to minimizing waste production in processes, would maximize the economical use of waste materials and of products at the ends of their lives as inputs to other processes and industries". The first full realization of an IS has been located in Kalundborg in Denmark to reduce costs and lessen environmental impact on the community[12]. Since the effective implementation of the 50-year-old Kalundborg project, the researches on industrial symbiosis have exploded[13]. However, few works consider the operational level like replenishment and scheduling activities[14].

In this work, we focus on a simple IS between three types of companies with symbiotic and reverse flows between them. The flows analysed could be easily extended to more firms without loss of information. The considered flows include a direct market, a reverse logistic and a second-hand market, which will be feed after no or few modifications on returned products. The symbiotic and reverse flows are established, but we do not consider times (lead, transportation, collection, filing, etc). The objective of the paper is to define the replenishment of inventories (for all products and waste) based on an echelon base stock strategy. This work is an extension of the work presented by Hennequin *et al.*[15] by considering stochastic and continuous flows, known as the stochastic fluid model. This model offers an interesting way to reduce the complexity inherent to discrete models by approximating the discrete material flows with continuous material flows[16]. In the base stock strategy, when a demand appears, the information is forwarded to all stages simultaneously[17].

The objective will be to find the base stocks (or order-up-to-level) that minimize the long run expected average total cost during the time horizon. This cost is composed by economic, environmental and social impacts for both manufacturing/remanufacturing products and waste. To find directly the sample derivative of this cost function in function of the base stock levels, we will derive information coming from a nominal trajectory compared with a perturbated trajectory (with a very small perturbation). The basic idea is that a single experiment could contain much more information about the system than conventional simulations utilize in their output analysis, including gradients. This sample path gradient estimation and optimization is known as the infinitesimal perturbation analysis (IPA) technique. IPA were first developed for study of discrete event dynamic systems throughput optimization[18]. Since then, the technique has been widely used for transfer lines and buffer strategies but also for telecommunication networks[19].

The paper is organized as follows. In the following section, we will study the related literature to outline the importance of developing new strategies within a sustainable framework while ensuring effective results. In the third section, the system under consideration is presented as well as the different considered material flows. In the fourth section, the definition of the objective function is given. In the fifth section, an infinitesimal perturbation analysis is conducted to compute the sensitivity of chosen performance measure, the echelon base stock policy, with respect to the system parameters and propose gradient estimators with respect to minimal total cost by a single simulation run. Then, we conduct a numerical application to estimate the base stock levels. The results are compared with other production policiy and with mathematical programming (with a discrete approach). Finally, we present conclusions and future developments related to our work.

2. Related literature and work positioning

Since the publication by the United Nations of "Our common future" report[20], many governments in the world have adopted strategies to strongly encourage individuals and companies to reduce their environmental and social impacts[24]. The sustainability concept, a very long-term notion, has been simplified and transcribed by the fact of acting essentially on three pillars: economy, environment and society[25]. However, generally, the sustainability debate focuses on reducing costs while minimizing environmental impacts, mainly a reduction in emissions of greenhouse gases[23], energy consumption and pollutants[4]. Social aspects of sustainability have primarily been discussed in terms of being a cause for, or possibly a solution to, environmental problems, rather than something that deserves attention as a sustainability component in its own right[24]. However, the social aspects of sustainability are increasingly being recognised as important, and a vast amount of literature discussing social sustainability has emerged within different fields such as sociology, ecology, finance, etc.

To limit their impacts on environment and society, companies need to optimize their operations decisions in sourcing, production, transportation, inventory, distribution etc. [25]. These strategies should aim to steer the planning process towards sustainable development over the entire life cycle of the products/services offered, via a set of objectives taken. More precisely, firms should develop new technologies, processes and management strategies, products and services to ensure economic profits while minimizing their negative impacts[26]. Among the different developments proposed, the optimal management of flows (materials, energy as well as information flows) can be an interesting way to act, allowing to get closer to nature[14]. From the different methods developed, reverse logistics flows (such as product returns after use) as well as flows that are part of an industrial symbiosis (named in this work as symbiotic logistics flows) seem to be efficient methods that also allow different actors in the supply chain (producers, consumers and suppliers) to work together (see Figure 1). It also permits limiting the amount of material used and stored (inside the manufacturing processes and the inventory levels in the different echelons).

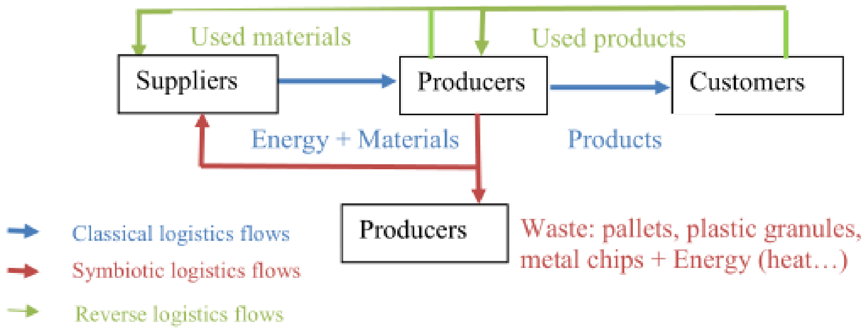

Figure 1 Logistics flows of materials and energy.

The number of publications concerning reverse flows are currently close to 1500 references during one month (main publishers of referenced journals, and almost 210 for only Taylor and Francis). For industrial symbiosis (IS) and symbiotic flows, the number of studies is lower (about 500 in one-month main publishers of referenced journals, and 50 in Taylor and Francis). In general, studies on IS are more concerned with its strategic or tactical implementation, with a study of the flows and interactions between actors, the network making up the IS and the ecopark[14]. For reverse flows, the studies could be dedicated to all part of business strategy and efficient management of upstream and downstream industrial processes[27], at strategic, tactical but also operational levels. However, for each flow, the recovery, sorting and sometimes the return in good condition or in a condition that is usable by the customer (second hand market or company requiring raw materials) is necessary. Recovery and sorting activities depends on the kind of collected waste and products.

However, to the best of our knowledge, there is no study on a common strategy. Based on this observation, we propose in this work to study these activities of material recovery, which can include water, and energy, from the point of view of producers. We then seek to define a production policy that considers the 3 pillars of sustainable development, i.e. economic (mainly production and inventory costs), environmental (carbon emissions) and social (hardship cost and employability benefits). This production policy will be based on inventory levels to minimize if possible the quantity of stored materials because in practice these are the most commonly policies[28]. The main problem concerning inventory control and production planning concerns information about the due date and the quantity of demands[29]. The chosen replenishment policy is the well-known base-stock policy will allow us to control the global flow of the symbiotic and reverse logistics flows based on the customers' demand information[17]. In this policy, a nominal inventory is associated with each stage. The objective is then to keep the inventory position of each stage equal to the level of its nominal inventory level. Furthermore, the base stock policy is appealingly simple to define and use, and it has been proven that it is optimal for systems without lead times[30] and suboptimal for different types of lead time and specificities[17]. The dynamic nature of the requirements and collection of finished products and waste generated for our considered system makes the solution procedure more complex in terms of computational effort. Furthermore, manufacturing and logistics activities may be difficult to study since the parameters cannot be easily obtained due to random influence and uncertainties.

Various uncertainties could be considered for symbiotic and reverse flows coming from the random flows of returns (quantity but also quality since value decay could play an important part in decision-making) to the demands (finished and second-hand finished products), with possible disruptions throughout the supply, manufacturing/remanufacturing and distribution processes[31]. We do not consider lead times, delivery times, etc., but delays may also occur on each stage and during transport. To simplify the analysis and the definition of strategies, some assumptions are commonly made to limit complexity and facilitate the decision-making. For the random reverse flows, generally researchers consider that the quantity of returned finished products is independent of sales. Indeed, real data are hard to collect, and the first steps towards reverse logistics flows are recent, dating back to 2004 with Dell. The quality of returns is often variable, depending not only on the type of product, but also on its use, and therefore on the type of customer[32]. Since then, an often-used hypothesis is that the process is memoryless and finite. In any case, quality is difficult to estimate[33], and in this case fuzzy approaches may be of interest. But whatever the model defined to take this type of uncertainty into account, not all collected products can be refurbished or resold without further processing. It has to be noticed that some studies have focused on finished products return after use, but usually in an end-of-life and closed loop context[34]. Concerning the demands, such as for the quantity of returned products, the need for second-hand products is difficult to define due to a lack of data and tractability. Some assumptions linked with irreducibility in case of queuing networks and finite Markov chains or processes could be defined. Whatever made assumptions, it is clear that the costs of managing returns depends on the models used[35].

Since for reverse and symbiotic flows, uncertainties have significant impacts on manufacturing/remanufacturing activities, forecasting of return rates, and inventory management[36-38], the choice of a resolution method could be crucial and could affect the obtained results. Generally, at an operational level (more specifically for reverse flows since few works have been done for symbiotic flows), researchers define mathematical models coupled with simulation schemes to evaluate the behaviour of the entire system and to derive strategies[39]. The mathematical formulation depends on the desired objectives and performances. For our case, the main existing works concern mathematical programming[40], lot sizing approaches[41] and multi-criteria decision making[42] with for each of them different kinds of considered uncertainties linked with parameters and variables and/or resolution methods including fuzzy tools[38]. However, the proposed models do not always integrate dynamical aspects and could not solve large-scale problems. From a sensitivity approach, to optimize these kinds of problems with uncertainties, the basic idea is to learn how to take decisions by observing and analysing the current behaviour of the system. Since the parameters in the modelled system represent quantities that can suffer from small errors, it is natural to analyse how the performance measures are affected by small changes in the parameters. Indeed, the effect of any change in the structure or parameters of a system can be decomposed into the effects of many jumps among states (or many perturbations). We then talk about perturbation analysis[43]. When an infinitesimally small perturbation is applied to a parameter, the order of occurrence of events is not changed. The method is known as the infinitesimal perturbation analysis (IPA) and allows to estimate the gradients of a stochastic variable on function of parameters of interest and can be used in stochastic optimization algorithms to determine the optimal parameter setting. In most cases, it can be shown that if the performance measure is continuous, the estimate of this performance measure is unbiased. In other words, the unbiasedness of this estimator corresponds to saying that between the nominal and perturbed trajectories of the performance measure no significant deviation (or bias) occurs. This means that when the derivative of the performance measure is estimated over a sufficiently long-time horizon, it can be approximated to the expected value of the performance measure. The conditions impartiality and convergence of the gradients are then obtained and can be applied to determined gradient estimators and thus used in an optimization algorithm. The main advantage is then to be able to compute optimal values without the help of all information and without long computation times. When the two conditions of impartiality and convergence can no longer be verified, other methods of perturbation analysis must be developed[44]. Initially developed for queuing networks[43], IPA has been applied for Markov processes. Suri and Fu[45] optimized the throughput of transfer lines and found gradient estimators with respect to maximal production rates for a general stochastic Markovian process. A different application[46] of this method has been studied with the determination of the gradients of the performance measures with respect to the production control parameters (case of the separation point). The dynamics of the system was represented by a stochastic fluid model[47] which allowed the authors to find the optimal parameters without using a detailed discrete event model. Indeed, the application of the IPA method to stochastic fluid models has been shown to produce better estimators for control and optimization than for performance evaluation, since it is possible to accurately determine the optimal parameters[48]. Woerner *et al.* [49] study the joint optimization of capacity and safety stock allocation in assembly systems with base-stock policies and periodic review by introducing a set of convex approximations. They then analytically compute sample path derivatives via infinitesimal perturbation analysis. In this paper, we apply the IPA technique to estimate the optimal level of base stocks for an industrial symbiosis system with reverse and symbiotic flows. The base stock levels are defined for finished products, for second-hand market and resale of waste.

In what follows, we present in details our considered system, we give the notations and the dynamics of the system based on stochastic fluid model and chosen policy.

3. Problem description

In this work, we do not consider a complete IS composed of various stakeholders and flows/interactions between them, but only the link between two companies to highlight the principle of operational symbiotic flow. The first one is the main company for which the production strategy, linked to the level of stocks, must be dimensioned. This company generates also waste that can be reused as input material for another company.

Similarly, for reverse logistics flows, we consider two types of customers, a direct customer of the main company consuming finished products. After use, this customer will return the used finished products via a collection system not detailed in this work. The used finished products can then be sorted and, depending on their condition, they will be resold to a second customer (representing a second-hand market) or returned to the main company's plant to be refurbished and marketed either as remanufactured products (considered as new) or as reconditioned products to the "second-hand" customer. The final objective is not to clearly identify and quantify all elements and parameters of the system under consideration but only to study and quantify the replenishment policy in order to reduce costs considering internalized environment and social impacts. Our study is like a zoom of the system focusing on the main company (see Figure 2). Indeed, for each environmental impact, we can define an associated cost (e.g. for greenhouse gas emissions a tax and/or penalty can be defined, for waste generated a cost directly linked to this waste can be defined, etc.). Similarly, for societal impacts, costs can be defined (e.g. the creation of new jobs with lower production costs because subsidies can be granted, etc.).

This study could be easily generalized to different finished products and waste, and more customers and other companies since the chosen model help us to consider flows and not quantities of products. However, if we add more waste and consumers, we should also add more constraints (and not only capacity constraints) and the number of variables will be huge which will complexify the simulation.

In this paper, we consider a continuous stochastic fluid model (SFM) to describe the overall dynamics of the system. The SFM will allow us to capture the main dynamics by accurately approximating this system which is in essence stochastic since different hazards can occur. The interest of using SFM is that it could easily integrate delays[50] and then integrate lead times, variability and uncertainties on lead times (not done in this work).

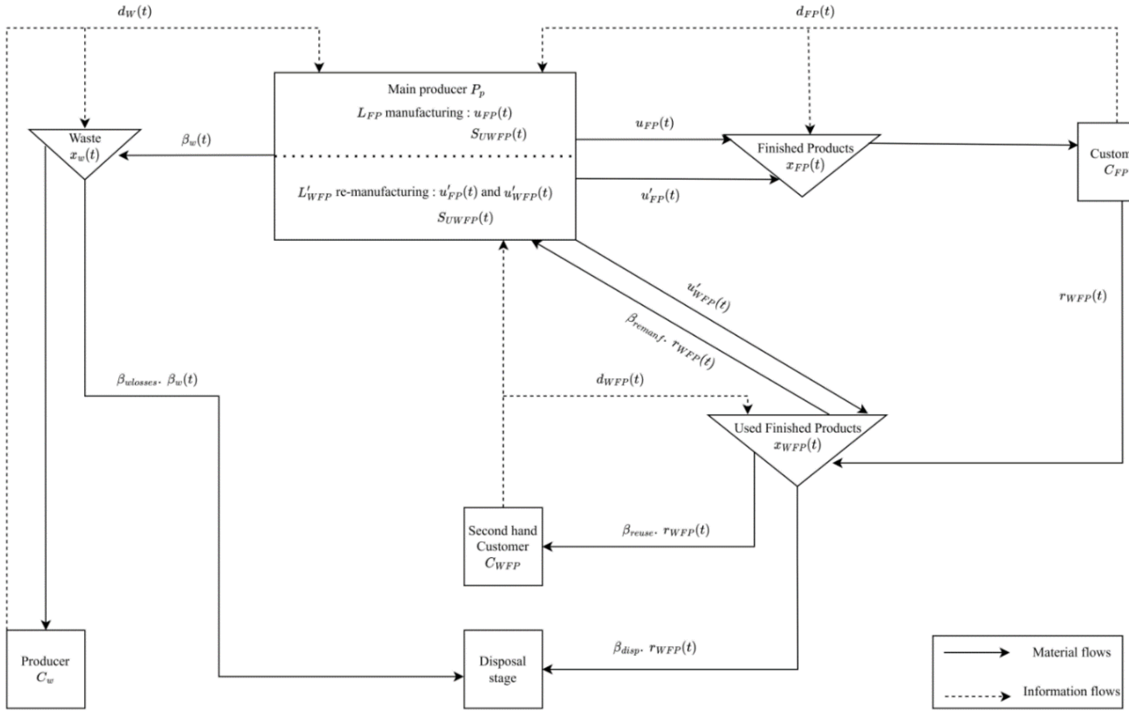

Figure 2 Considered system.

The main company is named P_P. It produces the finished products denoted *FP* in two different manufacturing lines: L_{FP} which produces finished products and L'_{FP} which remanufactures the collected used finished products. The finished products are stored in a finished products warehouse (denoted X_{FP}) of the main company P_P. The finished products are consumed by the customer C_{FP}. After use, these finished products are returned and stored in a dedicated warehouse, the wasted finished products warehouse X_{WFP}. It has to be noticed that for simplification we only consider the used finished products, denoted *WFP* coming from C_{FP}. The customer of finished products C_{FP} has an uncertain demand denoted as $d_{FP}(t)$. This uncertain demand of finished products is supposed to be independent from the symbiotic and reverse flows. In reality, the creation of a second-hand market can impact the initial demand of finished products, but this is not always the case. For example, in luxury industry -watches, or haute couture clothes, etc.- the customers of finished products are really different from the second-hand market. The reverse flow of used finished products obtained by the collection system is given by $r_{WFP}(t)$ with $r_{WFP}(t) \leq d_{FP}(t)$, $r_{WFP}(t)$ (also uncertain). We suppose that this demand is also independent from the first one (for the same reason). In warehouse X_{WFP}, wasted finished products are sorted into three categories:

- products that are no longer usable and are sent to a disposal stage -valorisation, recycle or landfill- with a flow equals to $\beta_{disp}.\ r_{WFP}(t)$,
- products that are reusable as is for a second-hand market represented by the customer C_{WFP} with a flow equals to $\beta_{reusable}.\ r_{WFP}(t)$,
- products that need to be reconditioned and send back to the main company with a flow equals to $\beta_{remanuf}.\ r_{WFP}(t)$. After reconditioning, these products could be sold to C_{WFP} (the second-hand market) or remanufactured if their condition allows it and could be sold to C_{FP}. Indeed, in function of their state after use by customer C_{FP}, these products could be remanufactured to an "as good as new" state or repackaged. In the case of remanufacturing to an "as good as new" state, we assume that the production of finished products of line L'_{WFP} is lower than that of line L_{FP} (due to losses and resales). Therefore, we define the finished product output of line L'_{WFP} as a percentage (defined as α'_{FP}) of the production rate of line L_{FP}. The production speed of line L_{FP} is equal to $u_{FP}(t)$ and the production speed of line L_{WFP} is equal to $u_{WFP}(t)$.

When the main producer P_P manufactures, it also generates waste which could be heat, water and material. In this paper for lake of simplicity, we consider that the flow of generated waste is equal to $\beta_W.(u_{FP}(t)+ u_{WFP}(t))$ with $\beta_W \leq 1$.This waste is stored in a warehouse denoted X_W. This waste could be sold to the customer C_W and corresponds to the symbiotic flow. The demand of this customer is also uncertain and given by $d_W(t)$. Concerning symbiotic flows, the demand is clearly independent from the demand of finished products since the market (products and costumers) is totally different.

The notation variables are given in Table 1. The parameters (costs and percentages) will be given in Table 2, in section 4.

Table 1 Variables of the considered system.

P_P	Main producer
L_{FP}	Production line of finished products
$u_{FP}(t)$	Production speed at time t of line L_{FP}
U_{FP}	Maximum production speed of line L_{FP}
$S_{UFP}(t)$	State at time t of the line L_{FP}: $\begin{cases} 1 \; if \; the \; line \; is \; able \; to \; produce \\ 0 \; else \end{cases}$
L'_{WFP}	Remanufacturing line of used finished products with 2 options: remanufacting to a "Good as new" state or repackaging for the 2nd-hand market
$u'_{FP}(t)$	Production speed in case of remanufacturing at time t of line L'_{WFP}
U'_{FP}	Maximum production speed for remanufacturing activities of line L'_{WFP}
$u'_{WFP}(t)$	Production speed in case of repackaging at time t of line L'_{WFP}
U'_{WFP}	Maximum production speed for repackaging activities of line L'_{WFP}
$S_{U'WFP}(t)$	State at time t of the line L'_{WFP}: $\begin{cases} 1 \; if \; the \; line \; is \; able \; to \; produce \\ 0 \; else \end{cases}$
$\beta_W(t)$	Waste recovery level with $0 \leq \beta_W(t) \leq \beta_{Wmax}$
β_{Wmax}	Maximum level of waste generated by the main company in the 2 considered lines considered
$\beta_{losses}(t)$	Losses (sent to the disposal stage) generated during the production and the sorting by the main company P_P
X_{FP}	Warehouse of finished products
$x_{FP}(t)$	Inventory level at time t of finished products
B_{FP}	Base-stock leve lof warehouse X_{FP}
$d_{FP}(t)$	Demand at time t of the customer C_{FP}
X_{WFP}	Warehouse of used finished products
$x_{WFP}(t)$	Inventory level at time t of used finished products
B_{WFP}	Base-stock leve lof warehouse X_{WFP}
$d_{WFP}(t)$	Demand at time t of the customer C_{WFP}
$r_{WFP}(t)$	Flow of returned used finished products at time t from the customer C_{FP} to the warehouse X_{WFP}
X_W	Warehouse of waste
$x_W(t)$	Inventory level at time t of waste
B_W	Base-stock leve lof warehouse X_W
$d_W(t)$	Demand at time t of the customer C_W

Then, the inventory level of finished products, denoted $x_{FP}(t)$, is given by:

$$(t) = \int_0^t [u_{FP}(s) + u'_{FP}(s) - d_{FP}(s)].ds \quad \text{(Eq.1)}$$

The inventory level of used finished products, denoted $x_{WFP}(t)$, is given by:

$$x_{WFP}(t) = \int_0^t [u'_{WFP}(s) + r_{WFP}(s)) - d_{WFP}(s)].ds \quad \text{(Eq.2)}$$

The inventory level of waste, denoted $x_W(t)$, is given by:

$$x_W(t) = \int_0^t [\beta_W(t).(1 - \beta_{Wlosses}) - d_W(s)].ds \quad \text{(Eq.3)}$$

The total losses are then given at time *t* by: $\beta_{losses}(t) = \beta_{Wlosses}.\beta_W(t) + \beta_{disp}.r_{WFP}(t)$.

For the used and collected finished products, the flows are given by:

$$r_{WFP}(t + \tau') = \beta_{remanuf}.r_{WFP}(t) + \beta_{reusable}.r_{WFP}(t) + \beta_{disp}.r_{WFP}(t)$$

The production strategy is based on a base stock echelon policy which means that all actors have the same information concerning the demands and that the level of inventories are limited by the base stock level for each considered warehouse. It is a pull control policy that triggers manufacturing orders in response to the arrival of demands. The production policy depends of the fact that it is necessary to produce and that the line can produce or not (because of possible disruptions, failures). The uncertainties in this case are represented by a memoryless model. This means that if ever production stops during manufacturing, the products must be resumed from the start (case of the chemical industry for example). For this replenishment policy, we define the base stock level in the finished products warehouse as B_{FP}:

$$B_{FP} = x_{FP}(t) - dFP(t) \quad \text{(Eq.4)}$$

The base stock in the used finished products warehouse is given by:

$$B_{WFP} = x_{WFP}(t) - dWFP(t) \quad \text{(Eq.5)}$$

The base stock in the waste warehouse is given by:

$$B_W = x_W(t) - dW(t) \quad \text{(Eq.6)}$$

Then, the production policy for the line L_{FP} of the main producer is given by:

$$u_{FP}(t) = \begin{cases} 0 \; if \; S_{UFP}(t) = 0 \\ U_{FP} \; if \; S_{UFP}(t) = 1 \; and \; S'_{UWFP}(t) = 0 \; and \; B_{FP} + d_{FP}(t) > x_{FP}(t) \\ U_{FP}.(1 - \propto'_{FP}) \; if \; S_{UFP}(t) = 1 \; and \; S'_{UWFP}(t) = 1 \; and \; B_{FP} + d_{FP}(t) > x_{FP}(t) \\ min(U_{FP}, d_{FP}(t)) \; if \; S_{UFP}(t) = 1 \; and \; S'_{UWFP}(t) = 0 \; and \; B_{FP} + d_{FP}(t) \le x_{FP}(t) \\ min(U_{FP}.(1 - \propto'_{FP}), d_{FP}(t)) \; if \; S_{UFP}(t) = 1 \; and \; S'_{UWFP}(t) = 1 \; and \; B_{FP} + d_{FP}(t) \le x_{FP}(t) \end{cases} \quad \text{(Eq.7)}$$

The production policies for the line L'_{WFP} of the main producer are given by:

$$u'_{FP}(t) = \begin{cases} 0 \; if \; S'_{UWFP}(t) = 0 \\ U'_{FP} \; if \; S'_{UWFP}(t) = 1 \; and \; S_{UFP}(t) = 0 \; and \; B_{FP} + d_{FP}(t) > x_{FP}(t) \\ min(U_{FP}.(1 - \propto'_{FP}), d_{FP}(t)) \; if S'_{UWFP}(t) = 1 \; and \; S_{UFP}(t) = 1 \; and \; B_{FP} + d_{FP}(t) > x_{FP}(t) \\ min(U'_{FP}, d_{FP}(t)) \; if \; S'_{UWFP}(t) = 1 \; and \; S_{UFP}(t) = 0 \; and \; B_{FP} + d_{FP}(t) \le x_{FP}(t) \\ min(U_{FP}.(1 - \propto'_{FP}), d_{FP}(t)) \; if \; S_{UFP}(t) = 1 \; and \; S'_{UWFP}(t) = 1 \; and \; B_{FP} + d_{FP}(t) \le x_{FP}(t) \end{cases} \quad \text{(Eq.8)}$$

$$u'_{WFP}(t) = \begin{cases} 0 \; if \; S'_{UWFP}(t) = 0 \\ U'_{WFP} \; if S'_{UWFP}(t) = 1 \; and \; B_{WFP} + d_{WFP}(t) > x_{WFP}(t) \\ min(U'_{WFP}, d_{WFP}(t)) \; if S'_{UWFP}(t) = 1 \; and \; B_{WFP} + d_{WFP}(t) \le x_{WFP}(t) \end{cases} \quad \text{(Eq.9)}$$

For the generation of waste, the two lines produce waste. We assume for simplicity that this waste is of the same type. Moreover, the waste generation is assumed to be proportional to what is produced. We could consider a model with random waste generation but most often it is known. The main company can choose to treat this waste in order to resell it or not to treat it. In this case a loss is defined, with an associated cost, linked to the fact of having to pay for the treatment of this waste. In the case where the waste is resold to another company, the main company may wish to shape it and may decide to do so or not, depending on the defined base stock strategy. Then, the waste recovery level is defined by:

$$\beta_W(t) = \begin{cases} 0 \; if \; S_{UFP}(t) = 0 \; and \; S'_{UWFP}(t) = 0 \\ \beta_{Wmax} \; if S_{UFP}(t) = 1 \; and \; S'_{UWFP}(t) = 1 \; and \; B_W + d_W(t) > x_W(t) \\ \beta_{W1} \; if S_{UFP}(t) = 1 \; and \; S'_{UWFP}(t) = 0 \; and \; B_W + d_W(t) > x_W(t) \\ \beta_{W2} \; if S_{UFP}(t) = 0 \; and \; S'_{UWFP}(t) = 1 \; and \; B_W + d_W(t) > x_W(t) \\ min(\beta_{Wmax}, d_W(t)) \; if S_{UFP}(t) = 1 \; and \; S'_{UWFP}(t) = 1 \; and \; B_W + d_W(t) \leq x_W(t) \\ min(\beta_{W1}, d_W(t)) \; if S_{UFP}(t) = 1 \; and \; S'_{UWFP}(t) = 0 \; and \; B_W + d_W(t) \leq x_W(t) \\ min(\beta_{W2}, d_W(t)) \; if S_{UFP}(t) = 0 \; and \; S'_{UWFP}(t) = 1 \; and \; B_W + d_W(t) \leq x_W(t) \end{cases} \quad \text{(Eq.10)}$$

For all considered stages, we consider social and environmental impacts. For transportation, we could have emissions of greenhouse gas and emissions of fine particles causing cancer and/or noise from the means of transport and/or road traffic congestions. For production stage, we consider also emissions of greenhouse gas and of pollutants and work accidents and other musculoskeletal disorders and in the inventory stage we integrate also emissions of greenhouse gas and of pollutants and work accidents and other musculoskeletal disorders. These impacts will be considered as internalized economic costs integrated in the objective function to minimize. The details of the objective function are given in the next section

4. Optimal replenishment policy for reverse and symbiotic flows

The objective of the present work is to obtain an optimal replenishment policy for the main company P_P. This policy should help to define the production and inventory management system trying to minimize costs. In this paper, we do not consider the service level as another objective to maximize because for industrial symbiosis, the customers may have needs that may not be fully met by the company P_P. The parameters are given in Table 3.

Table 3 Parameters of the considered system.

β_{disp}	Percentage of collected used finished products which will be sent to the disposal stage
$\beta_{resuable}$	Percentage of collected used finished products which will be sent to the customer C_{WFP}
$\beta_{remanuf}$	Percentage of collected used finished products which will be sent to the planto f the main company P_P
$\beta_{Wlosses}$	Percentage of generated waste which will be sent to the disposal stage
β_{W1}	Percentage of waste generated during the production by the line L_{FP} of the main company P_P
β_{W2}	Percentage of waste generated during the production by the line L'_{WFP} of the main company P_P
α'_{FP}	Percentage of production speed of the line L_{FP} for remanufacturing activities
$I(x_{FP}(t))$	Total inventory cost for finished products at time t
$I'(x_{WFP}(t))$	Total inventory cost for used finished products at time t
$I''(x_W(t))$	Total inventory cost for waste at time t
$P(x_{FP}(t))$	Total production cost for line L_{FP} at time t
$P'(x_{WFP}(t))$	Total production cost for remanufacturing and line L'_{WFP} at time t
$P''(x_W(t))$	Total production cost for repackaging and line L'_{WFP} at time t
$T(x_{FP}(t), x_{WFP}(t))$	Total transportation cost at time t
$C_{env}(x_{FP}(t))$	Total environmental cost for finished products. This cost is composed by carbon taxes and waste disposal fees.
$C'_{env}(x_{WFP}(t))$	Total environmental cost for used finished products. This cost is composed by carbon taxes and transportation/collection costs.
$C_{soc}(x_{FP}(t))$	Total social cost for finished products. For example, costs of manpower in case of incident/accident.
$C'_{soc}(x_{WFP}(t))$	Total social cost for used finished products. This cost could be lesser than for finished products since some subsidies could be allocated.
$C_{lost}(\beta_{losses}(t))$	Total losses cost for disposal stage
$G(x_W(t))$	Total gain obtained by reselling the waste

We suppose that:

Assumption 1: The initial conditions for the implementation of an echelon base stock policy are, at the initial time (t=0), there is no production and the levels of the both stocks are equal to the corresponding base sock levels before the arrival of a demand[50].

Assumption 2: All costs at given time *t* are independent of the base stock levels except the inventory costs. This hypothesis could be restrictive because the production costs depend on the base stock strategy, but by making this hypothesis we first simplify the theoretical study (the resulting equations will be more simple to express) and avoid the non-ergodicity possible phenomena that could occur and that would no longer allow us to study the problem using the IPA approach (because in this case biases could occur between the nominal trajectory and the perturbed trajectory, even if the perturbation is infinitesimally small).

Assumption 3: For the same reason, we consider priorities between all possible events: i) arrival of a demand in the finished warehouse; ii) definition of $d_{FP}(t)$, $d_{WFP}(t)$ and $d_W(t)$; iii) production of finished products by lines L_{FP} and L'_{WFP} (we consider that lead times are null); iv) demand satisfaction in all warehouses; v) calculation of shortfall and inventory level the warehouses; and vi) calculation of economic costs (holding and backlog, production, transportation, environmental and social costs) and the objective function. Indeed, this hypothesis is necessary to apply the infinitesimal perturbation analysis and could be obtained by applying the Little's Law[51].

Furthermore, it has to be noticed in this paper we do not consider the non-quality returns or product recalls or readjustments.

The objective is then to minimize the total cost function, denoted by $C(x_{FP}(t), x_{WFP}(t), x_W(t))$. This cost function is given by:

$$C(x_{FP}(t), x_{WFP}(t), x_W(t)) = I(x_{FP}(t)) + I'(x_{WP}(t)) + I''(x_W(t)) + P(x_{FP}(t)) + P'(x_{WFP}(t)) + P''(x_W(t)) + T(x_{FP}(t), x_{WFP}(t)) + C_{env}(x_{FP}(t)) + C'_{env}(x_{WFP}(t)) + C_{soc}(x_{FP}(t)) + C'_{soc}(x_{WFP}(t)) + C_{lost}(\beta_{losses}(t)) - G(x_W(t)) \quad \text{(Eq.11)}$$

Where $I(x_{FP}(t))$ is the total warehouse cost for finished products (including holding and backlog costs, see equation (12) below), $I'(x_{WFP}(t))$ is the total warehouse cost for used finished products, $I''(x_W(t))$ is the total warehouse cost for used finished products, $P(x_{FP}(t))$ is the total production cost for finished products (line L_{FP}), $P'(x_{WPF}(t))$ is the total production cost for remanufacturing and $P''(x_W(t))$ total production cost for the repackaging of used finished products (line L'_{WFP}), $T(x_{FP}(t), x_{WFP}(t))$ is the total transportation cost for finished and used products (same cost since the companies are located in the same industrial park, $C_{env}(x_{FP}(t))$ is the total environmental cost and $C_{soc}(x_{FP}(t))$ the total social cost for finished products, $C'_{env}(x_{WP}(t))$ is the total environmental cost and $C'_{soc}(x_{WP}(t))$ the total social cost for used finished products. $G(x_W(t))$ is the total gain obtained with the waste (including environmental and social aspects).

$$I(x_{FP}(t)) = \begin{cases} i+ . x_{FP}(t) \ if \ x_{FP}(t) > 0 \\ i- . x_{FP}(t) \ if \ x_{FP}(t) \leq 0 \end{cases} \quad \text{(Eq.12)}$$

The average total cost over an infinite time horizon is given by the following equation, it is given as a function of the base stock levels.

$$L(x_{FP}(t), x_{WP}(t), x_W(t)) = \left[\lim_{T\to\infty} \left(\frac{1}{T} \left(\int_0^T C(x_{FP}(t), x_{WP}(t), x_W(t)) \right) . dt \right) \right] \quad \text{(Eq.13)}$$

From assumption 2, this total average cost is equal to:

$$L(t, \vec{B}) = \left[\lim_{T\to\infty} \left(\frac{1}{T} \left(\int_0^T I(s, \vec{B}) + I'(s, \vec{B}) + I''(s, \vec{B}) \right) . ds + A \right) \right] \quad \text{(Eq.14)}$$

With
$$\vec{B} = \begin{bmatrix} B_{FP} \\ B_{WP} \\ B_W \end{bmatrix} \quad \text{(Eq.15)}$$

And

$$A = P(x_{FP}(t)) + P'(x_{WFP}(t)) + P''(x_W(t)) + T(x_{FP}(t), x_{WFP}(t)) + C_{env}(x_{FP}(t)) + C'_{env}(x_{WFP}(t)) + C_{soc}(x_{FP}(t)) + C'_{soc}(x_{WFP}(t)) + C_{lost}(\beta_{losses}(t)) - G(x_W(t)) \quad \text{(Eq.16)}$$

In what follows, we will study the nominal and perturbed trajectories of inventories in the warehouses.

5. Infinitesimal perturbation analysis

This section is devoted to the study of the trajectories which are given in terms of decision variables for the problem. They correspond to the base stock level of each warehouse. For this purpose, we define a positive perturbation (but similar results are obtained with a negative perturbation) for each of the *FP* and *WFP* and waste inventories: Δ_{FP} for finished products, Δ_{WFP} for used finished products and Δ_W for waste. The nominal trajectories for inventories will be represented by $x_{FP}(t)$ for finished products, by $x_{WFP}(t)$ for used finished products and by $x_W(t)$ for waste. The perturbated trajectory for inventories will be represented by $x_{FP}{}^{\Delta}(t)$ for finished products, by $x_{WFP}{}^{\Delta}(t)$ for used finished products and by $x_W{}^{\Delta}(t)$ for waste.

We obtain the following lemmas. The proof of these lemmas is similar to the one given in[52]. It consists of the study of all possible cases on the time horizon.

Lemma 1: $x_{FP}(t) \le x_{FP}{}^{\Delta}(t) \le x_{FP}(t) + \Delta_{FP},\ \ \forall\, t.$

Lemma 2: $x_{WFP}(t) \le x_{WFP}{}^{\Delta}(t) \le x_{WP}(t) + \Delta_{WFP},\ \ \forall\, t.$

Lemma 3: $x_W(t) \le x_W{}^{\Delta}(t) \le x_W(t) + \Delta_W,\ \ \forall\, t.$

These lemmas highlight the fact that the disturbed path is bounded by the nominal path, which will allow to propose a simulation-based optimization. After studying the trajectories, we can define a gradient of the cost function.

For an infinitesimal perturbation Δ_{FP} on the base stock level for finished products, the inventory cost due to the perturbation can be expressed as:

$$I(x_{FP}(t)) = \begin{cases} i^{+}.\ \Delta_{FP}, & si\ \ \Delta_{FP} \ge 0 \\ i^{-}.\ \Delta_{FP}, & si\ \ \Delta_{FP} < 0. \end{cases} \quad \text{(Eq.17)}$$

Let $T1(B_{FP})$ be the total period during which the inventory level of finished products is positive, $T2(B_{FP})$ be the total period during which the inventory level of finished products is negative.

On the time horizon *T*, equation (17) can be written as:

$$\frac{1}{T}\int_0^T (I(x_{FP}(t)))dt = \frac{1}{T}\left[\int_0^{T1(B_{FP})} (i^{+}.\ \Delta_{FP})dt - \int_0^{T2(B_{FP})} (i^{-}.\ \Delta_{FP1})dt\right]$$

$$\frac{1}{T}\int_0^T (I(x_{FP}(t)))dt = \frac{\left[i^{+}.\ \Delta_{FP}.T1(B_{FP})\ - i^{-}.\ \Delta_{FP}.T2(B_{FP})\ \right]}{T}$$

We obtain same results for an infinitesimal perturbation Δ_{WFP} on the base stock level for used finished products and Δ_W on the base stock level for waste. Let $T3(B_{WFP})$ be the total period during which the inventory level of used finished products is positive and $T4(B_{WFP})$ the total period during which the inventory level of used finished products is negative. Let $T5(B_W)$ be the total period during which the inventory level of waste is positive and $T6(B_W)$ the total period during which the inventory level of waste is negative. We obtain the following equation for average perturbated total cost over a finite horizon:

$$L_T^{\Delta}(t, B_{FP} +\ \Delta_{FP}, B_{WFP} +\ \Delta_{WFP}, B_W +\ \Delta_W) = \quad L_T(t,\vec{B}) + \frac{T1(B_{FP}).i^{+}.\Delta_{FP} - T2(B_{FP}).i^{-}.\Delta_{FP} + T3(B_{WFP}).i^{+}.\Delta_{WFP} - T4(B_{WFP}).i^{-}.\Delta_{WFP} + T5(B_W).i^{+}.\Delta_W - T6(B_W).i^{-}.\Delta_W}{T} \quad \text{(Eq.18)}$$

We can then estimate the gradient from the trajectories:

$$\frac{\partial L_T(t,\vec{B})}{\partial \vec{B}} = \lim_{\Delta_i \to 0} \frac{L_T(t, B_{FP} + \Delta_{FP}, B_{WFP} + \Delta_{WFP}, B_W + \Delta_W) - L_T(t,\vec{B})}{\Delta_i} \quad \text{(Eq.19)}$$

Consequently, we obtain:

$$\frac{\partial L_T(t,\vec{B})}{\partial \vec{B}} = \frac{T1(B_{FP}).i^+ - T2(B_{FP}).i^- + T3(B_{WFP}).i^+ - T4(B_{WFP}).i^- + T5(B_W).i^+ - T6(B_W).i^-}{T}$$

Theorem 1: The estimator of the cost gradient is unbiased.

$$\frac{\partial E[L_T(t,\vec{B})]}{\partial \vec{B}} = E\left[\frac{\partial L_T(t,\vec{B})}{\partial \vec{B}}\right]. \quad \text{(Eq.20)}$$

Proof of Theorem 1

To prove this, we need to show that the following 2 conditions are satisfied. The first condition is to prove that the derivative of the cost function $\partial[L_T(t,\vec{B})]/\partial\vec{B}$ exists with a probability 1 $\forall\ \vec{B}$. To do this, we compute the derivative on the right and the derivative on the left, they must be equal. The second condition is to prove that the function $L_T(t,\vec{B})$ is Lipschitz continuous and has a Lipschitz constant with a finite first moment.

- First condition:

The right derivative is given by:

$$\lim_{\Delta_i \to 0^+} \frac{L_T(t, B_{FP} + \Delta_{FP}, B_{WFP} + \Delta_{WFP}, B_W + \Delta_W) - L_T(t,\vec{B})}{\Delta_i} = \frac{T1(B_{FP}).i^+ - T2(B_{FP}).i^- + T3(B_{WFP}).i^+ - T4(B_{WFP}).i^- + T5(B_W).i^+ - T6(B_W).i^-}{T} \quad \text{(Eq.21)}$$

The left derivative is given by:

$$\lim_{\Delta_i \to 0^-} \frac{L_T(t,\vec{B}) - L_T(t, B_{FP} + \Delta_{FP}, B_{WFP} + \Delta_{WFP}, B_W + \Delta_W)}{\Delta_i} = \frac{T1(B_{FP}).i^+ - T2(B_{FP}).i^- + T3(B_{WFP}).i^+ - T4(B_{WFP}).i^- + T5(B_W).i^+ - T6(B_W).i^-}{T} \quad \text{(Eq.22)}$$

These derivatives are equal $\forall\ \vec{B}$. We thus prove that the derivative of the cost function exists.

- Second condition:

To guarantee ergodicity we need to verify that the demand must be satisfied on average over the time horizon. This condition is guaranteed by assumption 3 and the definition of the chosen policy. Indeed, the chose replenishment policy consists in producing at the maximum capacity of the machine when the production is behind the demand and the stock level is not allowed to decrease without limit.

In what follows, we will use these results in a simulation algorithm allowing us to directly calculate the optimal values of the base stock levels starting from an initial trajectory.

6. Computational examples

In the previous section, we studied the trajectories and expressed the disturbed trajectory of the inventory levels in terms of the nominal trajectory. We have determined that the deviation between the nominal trajectory and the perturbed trajectory is less than or equal to the value of the defined perturbation (Lemmas 1-3). Indeed, as the obtained estimator of the average total cost is unbiased we can use these theoretical results in the simulation, considering them as valid.

We implement these theoretical results in a simulation-based algorithm to determine the base stock levels that minimize the average total cost. We consider a simulation time which is large enough to derive reliable values (we chose a number of times when the finished products line is not able to produce -due to breakdowns, incidents...- equals to 500 000). The simulation algorithm is based on the principle of discrete event simulation. The principle is to calculate the states of the system and the dates of the next events, in order to know the evolution of the system and then use these results in the simulation. The initial conditions respect the Assumption 1 and we generate a set of random variables that we will use for all the simulations in order to ensure the validity of the Assumption 3. The simulation parameters are defined in Table 3. The production states of the lines are given by the same exponential laws with the parameter equals to 0.05 if the line is in condition to produce, with the parameter equals to 0.01 if the line is not in condition to produce. The demand for finished products is given by an exponential law of parameter 8, the demand for used finished products is given by an exponential law of parameter 2 and the demand for waste is given by an exponential law of parameter 1. The flow of returned products is given by an exponential law of parameter 5. The other parameters are the same as the work done by Hennequin *et al.*[15].

Table 3 Simulation parameters of the considered system.

Parameters	Values	Parameters	Values
U_{FP}	50	β_{disp}	10%
U'_{FP}	10	$\beta_{reusable}$	60%
U'_{WFP}	10	$\beta_{remanuf}$	30%
β_{Wmax}	30	$\beta_{Wlosses}$	20%
β_{W2}	40%	β_{W1}	40%
i^+	10	i^-	200

We compare our chosen production policy to the hedging point policy[53]. The principle of the hedging point is to find an optimal inventory level h^* that minimizes costs (generally production costs) by integrating uncertainties (such as breakdowns). The hedging point strategy consists of producing at maximum capacity if the inventory level is below this hedging point, producing nothing if it is above this hedging point, and producing at the rate of demand if it is equal to this hedging point. We obtain the following results (see Table 4) with $B*_i$ the optimal value obtained for the base stock levels and h^*_i the hedging points. To obtain these values, we first calculate the total average cost in function of the base stock level for finished products (the other base stock levels are equal to 0), then calculate the total average cost in function of the base stock level for used finished products (with $B*_{FP}$=47.45) and finally calculate the total average cost in function of the base stock level for waste. We have the same principle with the hedging points.

Table 4 Optimal obtained results.

	$\boldsymbol{B*_{FP}}$	$\boldsymbol{B*_{WFP}}$	$\boldsymbol{B*_{W}}$
$B*_i$	47.45	22.13	13.87
Total average cost (M.U.)	32136.82 MU	29512.37 MU	28524.56 MU
Standard error	0.0219	0.036	0.0251
	$h*_{FP}$	$h*_{WFP}$	$h*_{W}$
h^*_i	49.38	24.74	14.66
Total average cost (M.U.)	34454.23 MU	31736.12 MU	30627.15 MU
Standard error	0.022	0.025	0.023

These results depend heavily on various parameters and more specifically on costs and maximum production capacities, but also on needs. Indeed, for base stock levels the demands are considered for all stages which permits reducing the total amount of products in warehouses. It has to be noticed that we chose values for maximum production capacities that allow to respond to the demands.

We will also study the evolution of base stock levels and hedging points in function of the environmental and social costs during production, and losses cost. The other parameters remain the same. In figure 3, we represent the evolution of base stock level for used finished products in function of the environmental cost. In figure 4 we represent the evolution of base stock level of waste in function of the losses cost and in figure 5 we represent the evolution of base stock level, for finished products in function of the inventory cost i^+.

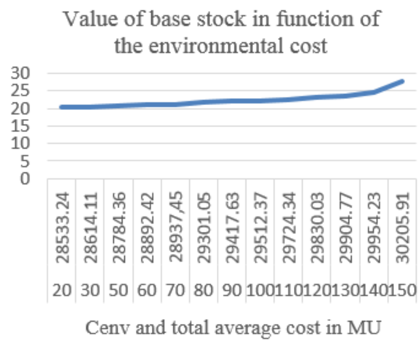

Figure 3 Evolution of the base stock level for used finished products in function of the environmental cost ($B*_{FP}$=47.45 and B_W =0).

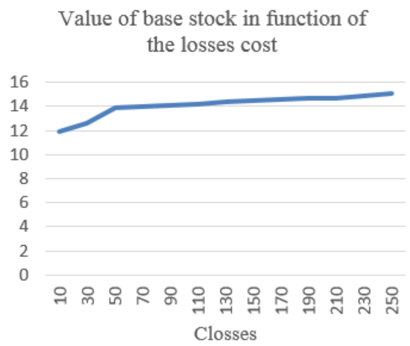

Figure 4 Evolution of the base stock level for waste in function of the losses cost ($B*_{FP}$=47.45 and $B*_{WFP}$=22.13).

It seems that the losses cost has a little impact on the value of the echelon base stock level for waste since the obtained values do not simply a linear regression like it seems to be for the first case with the echelon base stock for finished products. We do have a small inflection point that needs to be properly analysed for a cost equal to 50.

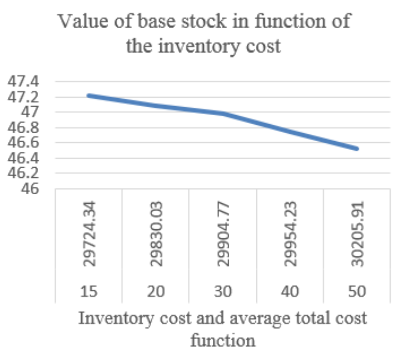

Figure 5 Evolution of the base stock level for finished products in function of the inventory cost (B_{WFP} =0and B_W =0).

Concerning the inventory cost, the echelon base stock will of course decrease if the value is more important. These results must be compared with the costs of backorder because if the ratio between the two increases or decreases the results obtained can be very different.

The next proposed simulation concerns the generation of random variables which do not necessarily allow for checking the ergodicity and this in order to extend the numerical results. More specifically, we consider that the states of the lines are defined by a Weibull law. The shape parameter and the scale parameter of the Weibull distribution are given by 2 and 100 respectively. The other parameters remain the same. We also compare, these results with discrete case and a mathematical programming approach simulated with uniform laws (see also this work[15]) in which we do not consider the waste flows and profits from the resale of waste). Then, we obtain the results given in Table 5.

Table 5 Optimal obtained results with a Weibull law for the states of the lines.

Continuous flows (Weibull law)	$\boldsymbol{B*_{FP}}$	$\boldsymbol{B*_{WFP}}$	$\boldsymbol{B*_{W}}$
$B*_i$ (SFM and IPA)	51.37	23.52	14.24
Total average cost (M.U.)	32435.43 MU	29948.78 MU	29647.37 MU
Standard error	0.009	0.024	0.017
	$h*_{FP}$	$h*_{WFP}$	$h*_{W}$
h^*_i (SFM and IPA)	53.47	26.23	16.52
Total average cost (M.U.)	34454.23 MU	31736.12 MU	30627.15 MU
Standard error	0.017	0.022	0.028
Discrete flows (Uniform laws)	$B*_{FP}$	$B*_{WFP}$	$B*_{W}$
$B*_i$ (mathematical prog.)	19	7	7
Total average cost (M.U.)	9316MU	9986MU	9169 MU
	$h*_{FP}$	$h*_{WFP}$	$h*_{W}$
h^*_i (mathematical prog.)	41	8	6
Total average cost (M.U.)	10337.9 MU	10226.9 MU	10186.5 MU

To estimate the optimal base stock levels, we apply the same strategy as before. It seems that our results are always verified even when the ergodicity is no longer totally guaranteed. But in this case, the proof of non-bias between the trajectories is not obvious. For the discrete case, since we calculate fewer points, the cost function values are lower.

7. Conclusion

In this paper, we have proposed an extension of the work presented by Hennequin *et al.* [15] by considering on the one hand a stochastic fluid model, and on the other hand, the infinitesimal perturbation analysis approach in order to define an optimization-based simulation ensuring optimal results by proving that the perturbated trajectory is unbiased compared with the nominal trajectory if the perturbation is sufficiently small and if the ergodicity is ensured. To greatly simplify the presentation and the mathematical equations, we have assumed that the costs are independent of these base stock levels, which also ensures ergodicity and therefore the simplification of the theoretical study. However, this assumption can be lifted and similar results can be found (with slightly different demonstrations) without affecting the fact that the perturbed path is unbiased. It should be noted that the numerical results obtained depend very strongly on the values of the parameters and it would therefore be interesting to conduct a sensitivity analysis. This can also be done theoretically using the infinitesimal perturbation analysis. The novelty of the presented work lies in the choice of the proposed system allowing to study inverse and symbiotic logistic flows. The expected total average cost composed of the production and inventory costs, internalized environmental and societal costs as well as the losses associated with possible resales (second-hand market and industrial symbiosis).

The next step is to conduct a sensitivity analysis and thus identify key parameters to facilitate the implementation of symbiotic flows and reverse logistics. Based on these results, it will be easier to clearly define the production and inventory management strategy. Other policies should be implemented in order to identify the best operational strategy to define. We will then be able to integrate into our operational optimization scheme real data collected on an existing eco-park in order to show to the actors the interest in investing in an industrial symbiosis. To do so, we will have to consider in our model the different delays not considered here (these can be very easily integrated in the stochastic fluid model). However, it should be noted that the industrial symbiosis only works if all the actors are clearly committed, studies on their behavior and their impact on the management would also be useful. Game theory could be an interesting method to use for this. Furthermore, for optimization of random systems, it is generally assumed that the parameters associated with stochastic input processes are known.

Conflict of interest

The authors declare no conflict of interest.

References

1. O'Brien C. (2013). Fifty years of shifting paradigms, International Journal of Operation Research, 51(23–24), p. 6740–6745.
2. Turken, N., Cannataro, V., Geda, A. and Dixit, A. (2022). Nature inspired supply chain solutions: definitions, analogies, and future research directions, International Journal of Production Research, 58(15), p. 4689-4715.
3. Benyus, J. M. (2002). Biomimicry: Innovation Inspired by Nature, the first version of this book has been published in 1997, Harper Perennial Edition.
4. McKinnon, A., Browne, M., Whiteing, A., Piecyk M. (2015). Green logistics – Improving the environmental sustainability of logistics (3rd Edition), Kogan Page, Ireland, https://www.perlego.com/book/1589465/green-logistics-pdf.
5. Fahimnia, B. , Sarkis, J. and Davarzanic, H. (2015). Green supply chain management: a review and bibliometric analysis. International Journal of Production Economics, 162 (4), 101–114.
6. Hazen, B.T., Wu, Y., Cegielski, C.G., Jones-Farmer, L.A. and Hall, D. J. (2012). Consumer reactions to the adoption of green reverse logistics, The International Review of Retail, Distribution and Consumer Research, 22(4), p. 417-434.
7. Das, K. (2012). Integrating reverse logistics into the strategic planning of a supply chain, International Journal of Production Research, 50(5), p. 1438-1456.
8. Vahabzadeh, A. H. and Mohd Yusuff, R.B. (2015). A content Analysis in Reverse Logistics: A review, Journal of Statistics and Management Systems, 18(4), p. 329-379.
9. Chern, C.C., Chen, P.Y. and Huang, K.W. (2014). A production base-stock policy for recycling supply chain management in the presence of uncertainty, Computers & Industrial Engineering, 76 (2014), p. 193–203.
10. Turken, N. and Geda, A. (2020). Supply chain implications of industrial symbiosis: A review and avenues for future research, Resources, Conservation & Recycling, 161 (2020) 104974.
11. Frosch, R. A., (1992). Industrial Ecology: A philosophical Introduction, Proceedings of the National Academy of Sciences of the USA, 89, p. 800-803.
12. Scott Victor V. (2016). Kalundborg Symbiosis: fostering progressive innovation in environmental networks, Journal of Cleaner Production, 118, p. 65-77.
13. Yazan D. M. and Fraccascia, L. (2020). Sustainable operations of industrial symbiosis: an enterprise input-output model integrated by agent-based simulation, International Journal of Production Research, 58(2), p. 392-414.
14. Hennequin, S. and Roy, D. (2021). Past, Present and Future of Industrial Symbiosis (chapter 3), An Introduction to the Circular Economy, Series: Economic Issues, Problems and Perspectives, Pierfrancesco Morganti and Maria-Beatrice Coltelli (Editors), Nova Science Publisher.
15. Hennequin, S., Yazdani M. A. and Roy, D. (2022). Echelon base stock policy for symbiotic and reverse logistic flows, Proceedings of the 10th IFAC Conference on Manufacturing Modelling, Management and Control, June 22-24, 2022, Nantes, France.

16. Bean, N. G., O'Reilly, M. M. and Palmowski, Z. (2022). Matrix-analytic methods for the analysis of stochastic fluid-fluid models, Stochastic Models, 38(3), p. 416-461.
17. Barron, I. and Dreyfuss, M. (2021). A triple (S, s, ℓ)-thresholds base-stock policy subject to uncertainty environment, returns and order cancellations, Computers & Operations Research, 134 (2021) 105320.
18. Ho, Y.C. (1987). Performance evaluation and perturbation analysis of discrete systems: Perspective and open problems, IEEE Transactions on Automatic Control AC-32, p. 563-572.
19. Cui, Z., Liu, Y. and Wang, R. (2022) Variance comparison between infinitesimal perturbation analysis and likelihood ratio estimators to stochastic gradient, Operations Research Letters, 50(2), p. 199-204.
20. Bruntdland, G.H. (1987). Our Common Future. Report of the World Commission on Environment and Development, published in 1987 by the United Nations, http://www.un-documents.net/our-common-future.pdf
21. Schernewski, G., Schönwald, S., Katarzyte, M., (2014). Application and evaluation of an indicator set to measure and promote sustainable development in coastal areas, Ocean & Coastal Management, 101 (2014) p. 2-13.
22. Redcliff, M., (2009). Sustainable development (1987 – 2005) – an oxymoron comes of age. Problemy Ekorozwoju, 4(1), p. 33-50.
23. Edgington, S. M., (1995). Industrial Ecology: Biotech's Role in Sustainable Development. Can an environmental concept rally business, government, and academia to work together for long-term economic growth? Nature Publishing Group http://www.nature.com/naturebiotechnology.
24. Littig, B. and Grieβler, E. (2005). Social sustainability: a catch word between political pragmatism and social theory, International Journal on Sustainable Development, 8(1-2), p. 65-79.
25. Hua, G., Cheng, T. C. E., & Wang, S. (2011). Managing carbon footprints in inventory management. International Journal of Production Economics, 132(2), p. 178-185.
26. Wang, J., Lim, M. K., Wang, C. and Lang Tseng, M. (2022). Comprehensive analysis of sustainable logistics and supply chain based on bibliometrics: overview, trends, challenges, and opportunities, International Journal of Logistics Research and Applications, published online.
27. Jraisat, L., Jreissat, M., Upadhyay A. and Kumar, A. (2023). Blockchain Technology: The Role of Integrated Reverse Supply Chain Networks in Sustainability, Supply Chain Forum: An International Journal, 24(1), p. 17-30.
28. Ghasemi, E., Lehoux, N. and Rönnqvist, M. (2022) Coordination, cooperation, and collaboration in production-inventory systems: a systematic literature review, International Journal of Production Research, published online.
29. Wang, Y., Zhang, Y. and Luo, X. (2022b) Configuring information updating schemes of partial imperfect advance demand information in production-inventory systems, Journal of the Operational Research Society, published online.
30. Fleischmann, M. and Kuik, R. (2003). On optimal inventory control with independent stochastic item returns. European Journal of Operational Research, 151 (1), p. 25–37.
31. Lieckens, K. and Vandaele, N. (2012). Multi-level reverse logistics network design under uncertainty, International Journal of Production Research, 50(1), p. 23-40.
32. Ramezani, M., Bashiri, M. and Tavakkoli-Moghaddam, R. (2013). A new multi-objective stochastic model for a forward/reverse logistics design with responsiveness and quality level, Applied Mathematical Modelling, 37(1-2), p. 328-344.
33. Eslamipirharati, M., Jolai, F. and Aghsami, A. (2023). A Bi-objective two-stage stochastic optimization model for sustainable reverse supply chain network design under carbon tax policy and government subsidy considering product quality, Journal of Industrial and Production Engineering, published online.
34. Benedito, E. and Corominas, A. (2013). Optimal manufacturing policy in a reverse logistic system with

dependent stochastic returns and limited capacities, International Journal of Production Research, 51(1), p. 189-201.

35. Wang, X., Zhao, M. and He, H. (2018). Reverse logistic network optimization research for sharing bikes, Procedia Computer Science 126, p. 1693-1703.
36. Fleischmann, M., Krikke, H. R., Dekker, R. and Flapper, S. D. P. (2000). A Characterisation of Logistics Networks for Product Recovery, Omega, 28, p. 653–666.
37. Ilgin, M. A. and Gupta, S. M. (2013). Reverse Supply Chains: Issues and Analysis, edited by Gupta, S., In Chapter: Reverse Logistics 18, p. 1-60, FL, USA: CRC PRESS.
38. Kazemi, N., Modak, N. M. and Govindan, K. (2019). A review of reverse logistics and closed loop supply chain management studies published in IJPR: a bibliometric and content analysis, International Journal of Production Research, 57(15-16), p. 4937-4960.
39. Abid, S. and Mhada, F. Z. (2021). Simulation optimisation methods applied in reverse logistics: a systematic review, International Journal of Sustainable Engineering, 14(6), p. 1463-1483.
40. Hatefi, S.M., Jolai, F., Torabi, S.A. and Tavakkoli-Moghaddam, R. (2015). A credibility-constrained programming for reliable forward–reverse logistics network design under uncertainty and facility disruptions, International Journal of Computer Integrated Manufacturing, 28(6), p. 664-678.
41. Alinoviy, A., Bottani, E. and Montanari, R. (2012). Reverse Logistics: a stochastic EOQ-based inventory control model for mixed manufacturing/remanufacturing systems with return policies, International Journal of Production Research, 50(5), p. 1243–1264.
42. Govindan, K., Diabat, A. and Madan Shankar, K. (2015). Analyzing the Drivers of Green Manufacturing with Fuzzy Approach, Journal of Cleaner Production, 96, p. 182–193.
43. Ho, Y. C., and Cao, X. (1983). Perturbation analysis and optimization of queueing networks. Journal of Optimization Theory and Applications, 40(4), p. 559-582.
44. Kassoul, K., Cheikhrouhou N. & Zufferey, N. (2021). Buffer allocation design for unreliable production lines using genetic algorithm and finite perturbation analysis, International Journal of Production Research, 60(10), published online.
45. Suri, R. and Fu, B.-R. (1994). On using continuous flow lines to model discrete production lines, Journal of Discrete Event Dynamic Systems: Theory and Applications, vol. 4, p. 127-169.
46. Yu, H., and Cassandras, C. G. (2004). Perturbation analysis for production control and optimization of manufacturing systems, Automatica, 40(6), p. 945-956.
47. Melamed, B., Pan, S., and Wardi, Y. (2007). Simulation of IPA gradients in hybrid network systems, Computers & Mathematics with Applications, 54(2), p. 161-182.
48. Markou, M. M., et Panayiotou, C. G. (2010). On-line control of the threshold policy parameter for multiclass systems, Automatica, 46(3), p. 528-536.
49. Woerner, S., Laumanns, M. and Wagner, S. M. (2017). Joint optimisation of capacity and safety stock allocation, International Journal of Production Research, 56(13), published online.
50. Boonlertvanich, K. (2005). Extended-CONWIP-Kanban system: control and performance analysis. PhD thesis, Georgia Institute of Technology, USA.
51. Xie, X., Hennequin, S. and Mourani, I. (2013). Perturbation Analysis and Optimization of Continuous Flow Transfer lines with Delay, International Journal of Production Research 51(23-24), p. 7250-7269.
52. Ramirez Restrepo, L. M., Hennequin, S. and Aguezzoul, A. (2016). Optimization of integrated preventive maintenance based on infinitesimal perturbation analysis. Computers & Industrial Engineering, 98, p. 470-482.
53. Akella, R. and Kumar, P. R. (1986). Optimal control of production rate in a failure prone manufacturing system, IEEE Transactions on Automatic Control, 31(2), pp. 116-126, 1986.