Industrial Management Advances (2025) Volume 3 Issue 1
doi: 10.59429/ima.v3i1.9928

ISSN: 3029-1674 (O)

RESEARCH ARTICLE

Exponential backordering inventory model addressing shortages in finite planning horizons

Renuka Sheshrao Namwad[1], Nitin Kumar Mishra[1*], Prerna Jain[2*] and Ranu[3]

[1] *Department of Mathematics, Lovely Professional University, Phagwara, 144411, Punjab, India*

[2] *Gitarattan International Business School, Guru Govind Singh Indraprastha University, Delhi, Zip-110078, INDIA*

[3] *Department of Mathematics, NIILM University, Haryana, Zip-136027, INDIA*

*** Corresponding authors:** Nitin Kumar Mishra, snitinmishra@gmail.com & Prerna Jain, prernajain0312@gmail.com

ABSTRACT

In today's highly dynamic and price-sensitive market environment, inventory management faces increasing challenges due to fluctuating demand and the need for efficient coordination between suppliers and retailers. Based on these premises, this paper develops an exponential complete backordering model of the inventory system that considers the shortages within a finite planning horizon and price-sensitive demand in the presence of linear trends. Included in considerations are variability of demand, backordering costs, and a demand-plus-price relationship modeled through exponential backordering functions taken into consideration. Supportive of MATHEMATICA 12, iterative calculations were performed and Python-based tools created graphs and heatmaps to clearly delineate the result. This would decrease the total variable cost for determining the optimal period of replenishment and shortage. A numerical example is then used to show the ability of the model in optimizing the inventory management, which in turn leads to reduced stockouts, good coordination among suppliers and retailers, and lower costs. The sensitivity analysis presented here is applied to represent the significance of parameters. This research now provides a strong structure for improvement in supply chain management involving dynamic demand environments, especially when price sensitivity and backordering largely influence the processes.

Keywords: Finite planning horizon; exponential backorder; shortage; complete backorder; price sensitive demand

1.Introduction

Efficient inventory management is fundamental to sustaining a competitive advantage in modern supply chain ecosystems. The complexity of demand patterns, coupled with uncertainties in supply, necessitates the development of sophisticated inventory models capable of addressing various challenges. Among these challenges, shortages of goods pose a significant concern, especially when considering both linear trends and price-sensitive demand dynamics across the planning horizon. In response to this multifaceted problem, this research endeavors to investigate an exponential complete backordering inventory model for a single-item, price-sensitive demand system under a deterministic finite planning horizon.

This model aims to comprehensively analyze the implications of shortages while accommodating both linear trends and price-sensitive demand throughout all cycles of the planning horizon. The concept of exponential backordering is essential because it realistically captures the cost escalation and customer dissatisfaction associated with increasing waiting times during stockouts. By integrating exponential

ARTICLE INFO
Received: 3 March 2025|Accepted: 29 April 2025|Available online: 21 May 2025

CITATION
R. S. Namwad;N. K. Mishra;P. Jain;Ranu. Exponential backordering inventory model addressing shortages in finite planning horizons.*Industrial Management Advances* 2025; 3(1): 9050.doi: 10.59429/ima.v3i1.9928

backordering principles and considering the nuances of price-sensitive demand, the proposed model seeks to provide insights into optimal inventory control strategies under such conditions. The significance of this research is underscored by its potential to enhance decision-making processes in inventory management, thereby minimizing costs and maximizing operational efficiency. The proposed model addresses critical factors such as demand variability, lead time, and backordering costs, offering a robust framework for managing inventory in environments characterized by fluctuating demand and supply uncertainties. Through a rigorous investigation of the proposed model, this study aims to contribute to the existing body of knowledge in inventory optimization, particularly in addressing shortages amidst dynamic demand and supply conditions.

This research employs advanced numerical iterative methods and leverages the computational capabilities of MATHEMATICA (version 12) to validate the model. The results demonstrate optimized total costs, taking into account exponential backorder rates and the price-sensitive nature of demand within a finite planning horizon. In light of these considerations, this paper outlines the objectives, methodology, results, implications, and future directions of the research. The overarching goal is to advance our understanding of inventory management in the face of evolving market dynamics, providing valuable insights for practitioners and decision-makers aiming to improve their inventory control mechanisms and maintain a competitive edge in an ever-changing business landscape.

2. Literature Review

The concept of exponential partial backordering has been widely studied in the inventory literature, providing a more realistic representation of customer behavior in situations where some customers are willing to wait for replenishment while others seek alternative sources. Researchers have explored various extensions and variations of this model, incorporating factors such as deteriorating items, time varying demand and production rates, pricing decisions, and different cost structures. Despite the extensive research in this area, there is still room for further exploration and refinement of these models to better capture the complexities of real-world inventory systems[1] offers a comprehensive and contemporary examination of inventory management techniques. With a strong emphasis on the integration of demand forecasting and inventory control, the book provides valuable insights for professionals and academics seeking to optimize supply chain efficiency[2] discusses advanced inventory management strategies, including backordering and the impact of demand variability on inventory decisions[3] examines the interplay between pricing strategies and inventory control, proposing models that accommodate demand elasticity and price fluctuations[4] delves into the theoretical foundations of supply chain management, offering insights into inventory models that address shortages and backordering[5] investigates inventory policies for perishable products with stochastic demand, providing insights into how inventory models can be adapted to manage products with varying shelf lives and demand patterns[6] discusses strategies for aligning supply and demand, emphasizing the need for flexible inventory models to manage shortages effectively[7] provides a comprehensive overview of production and inventory control, discussing strategies for managing backorders in complex supply chain networks[8] explores how analytics can improve inventory management, including the use of backordering policies to optimize supply chain performance[9] discusses the benefits of information sharing in supply chains, relevant to optimizing inventory levels and backordering strategies[10] The paper introduces a model on supply chain inventory replenishment addressing material degradation based on usage of green technologies, offering profits along with cost reduction as well as carbon emissions abatement through the use of an iterative numerical algorithm with theoretical insights and practical applications[11] provides a detailed examination of supply chain strategies, including inventory management and backordering policies to handle dynamic demand and supply conditions[12] explores how technology adoption, such as RFID, can improve inventory accuracy and management, including backordering[13] investigates inventory control policies for products with price sensitive demand, providing insights into how pricing strategies influence inventory levels and back-ordering decisions[14] examines how dynamic demand patterns influence inventory management strategies, including the role of backordering[15] discusses the role of contracts in capacity planning, including inventory management and backordering strategies with limited resources[16] explores the impact of agency issues on supply chain coordination, focusing on price-sensitivity impact on demand[17] examines how demand forecasting and lead times affect inventory management, emphasizing the need for robust backordering models[18] explores the causes and consequences of demand variability in supply chains, emphasizing the need for effective inventory models to manage backorders[19] explores the performance of various lot sizing rules in the presence of demand uncertainty, relevant to understanding the impact of backordering policies[15] provides a comprehensive review of single period inventory models, highlighting the challenges of managing backorders in such contexts[20] discusses the impact of setup costs on inventory decisions, relevant to the study of back-ordering models in contemporary supply chains[21] explores how production constraints impact inventory decisions, relevant to the study of backordering under capacity limitations[22] provides foundational insights into multi-echelon inventory systems, highlighting the importance of coordinated inventory policies[23] discusses common challenges in supply chain inventory management and proposes strategies to address these issues, including backordering policies[24] presents modifications to the Economic Order Quantity (EOQ) model, which are pertinent to understanding how setup cost reductions can affect back ordering strategies[25] provides insights into the role of stochastic processes in inventory management, highlighting the importance of probabilistic models in addressing supply chain uncertainties[26] explores how dynamic pricing strategies can influence inventory management and backordering decisions[8] discusses the benefits of information sharing in supply chains, relevant to optimizing inventory levels and backordering policies[27] explores how analytics can improve inventory management, including the use of backordering policies to optimize supply chain performance[28] investigates inventory policies for perishable products with stochastic demand, providing insights into how inventory models can be adapted to manage products with varying shelf lives and demand patterns.

3. Research Gap

Despite the significant contributions made by previous studies in modeling partial backordering, price-sensitive demand, and perishable inventory systems, limited attention has been given to exponential complete backordering models under a finite planning horizon that also incorporate price-sensitive demand with linear trends. Most existing models focus on either partial backordering or assume static demand, overlooking the complex interdependencies between pricing strategies, stockout costs, and replenishment timing in dynamic environments. Furthermore, the integration of exponential backordering cost structures with computational validation through tools like MATHEMATICA and Python-based visualizations remains underexplored in current literature. Therefore, this research addresses this gap by developing and analyzing an exponential complete backordering inventory model that reflects real-world complexities, aiming to improve total cost minimization and decision-making efficiency in modern supply chains.

4. Assumptions and Notations

4.1 Assumptions

a) The initial inventory level is zero.

b) The storage cost remains constant throughout.

c) The ordering cost is fixed and known in advance.

d) Shortages are allowed under a finite planning horizon.

e) Partial back ordering is applied, following an exponential function.

4.2. Notations

i. Fixed time horizon H.

ii. The demand rate is D and D(t)= a-bp.

iii. The amount that is carried per unit per order is denoted by r.

iv. O_r is the cost of replenishing or purchasing per order.

v. S is the shortage cost per unit time.

vi. I_j is the total inventory carried out during the interval $[t_j, s_j]$

vii. S_j is the total amount of shortages in the interval $[s_j, t_{j+1}]$.

viii. S_j denotes the time at which the inventory level reaches zero in the j^{th}replenishment cycle j= 1,2, 3, …., n.

ix. t_j is the j^{th} replenishment time j =1, 2, 3, ……n.

x. n is the number of orders during the time horizon H.

xi. D_l=a-bp-cp^2 is the price dependent quadratic backordering.

xii. Q is the total optimal order quantity during the planning horizon H.

xiii. $I_b$instantons shortage during the shortage period.

xiv. θ is an inventory dependent parameter.

xv. *B*(*t*)is the back ordering rate taken as an exponential function, $B(t) = \rho e^{-ct}$

4.3. Model formulation

The initial equation is given by, $\frac{dI_{j+1}(t)}{dt} + I_{j+1}(t) = D(t),\ t_j < t < s_{j+1}$ … (1)

Where, j=1, 2, 3, ……n_1

$$\frac{dI_{j+1}(t)}{dt} = -D(t) - I_{j+1}(t), t_j < t < s_{j+1} \quad \dots (2)$$

Consideringthe boundary conditions $I_{j+1}(s_j) = 0$

The solution of the equation (2) is,

$$I_{j+1}(t) = e^{-\alpha t}\int_t^{s_{j+1}} D(u)e^u\, du$$

$$I_{j+1}(t) = (a-bp)(e^{s_{j+1}-\alpha t} - e^{(1-\alpha)t}) \quad \dots (3)$$

During the shortage phase, the instantaneously arising shortages $I_b(t)$ is offered by,

$$I_b(t) = B(t)(t_{j+1}-s_j)$$

$$I_b(t) = \rho e^{-ct}(t_{j+1}-s_j) \quad \dots (4)$$

Considering the boundary conditions, $I_b(s_j)=0$

$$Q_{j+1}=I_{j+1}(t)=(a-bp)(e^{s_{j+1}-\alpha t_j}-e^{(1-\alpha)t_j})$$

Considering the reorganization of the ordering s_{j+1} can be given as,

$$S_{j+1}=\int_{s_j}^{t_j} I_b(t)dt$$

$$S_{j+1}=\int_{s_j}^{t_j} \rho e^{-ct}(t_{j+1}-s_j)$$

$$S_{j+1} = \frac{-\rho}{c}(t_{j+1}-s_j)(e^{-ct_j}-e^{-cs_j}) \qquad \ldots\ (5)$$

The entire purchase amount for a limited time frame of planning,

$$Q_{nt}=\sum_{j=1}^{n_1} Q_{j+1}=\sum_{j=1}^{n_1}(I_{j+1}+S_{j+1})$$

$$Q_{j+1}=\sum_{j=1}^{n_1}[(a-bp)\left(e^{(s_{j+1}-\alpha t_j)}-e^{(1-\alpha)t_j}\right)-\frac{\rho}{c}(t_{j+1}-s_j)(e^{-ct_j}-e^{-cs_j})] \qquad \ldots\ (6)$$

The total retailer cost over a specified time horizon is given by,

Total Cost= Re-supply expensess + Cost of retaining + Stocks + Purchacing Cost + Storage Cost

$$T_R\left(t_j,s_j,n_1\right)=n_1.O_r+\sum_{j=0}^{n_1-1} H\int_{t_j}^{s_{j+1}} I_{j+1}(t)dt+\sum_{j=0}^{n_1-1} W_h.Q_{j+1}+\sum_{j=0}^{n_1-1} S\int_{s_j}^{t_j} I_b(t)dt \qquad \ldots\ (7)$$

$$T_R\left(t_j,s_j,n_1\right)=n_1.O_r+\sum_{j=0}^{n_1-1} H\int_{t_j}^{s_{j+1}}(a-bp)\left(e^{s_{j+1}-\alpha t}-e^{(1-\alpha)t}\right)dt+$$

$$\sum_{j=0}^{n_1-1} W_h.\left[(a-bp)\left(e^{(s_{j+1}-\alpha t_j)}-e^{(1-\alpha)t_j}\right)\right]+\sum_{j=0}^{n_1-1} S\int_{s_j}^{t_j} \rho e^{-at}(t_{j+1}-s_j)dt$$

$$T_R\left(t_j,s_j,n_1\right)=n_1.O_r\sum_{j=0}^{n_1-1} H.(a-bp)\left[\frac{-1}{\alpha}(1-e^{s_{j+1}-\alpha t_j})-\frac{1}{(1-\alpha)}\left(e^{(1-\alpha)s_{j+1}}-e^{(1-\alpha)t_j}\right)\right]+$$

$$\sum_{j=0}^{n_1-1} W_h.\left[(a-bp)\left(e^{(s_{j+1}-\alpha t_j)}-e^{(1-\alpha)t_j}\right)\right]-\sum_{j=0}^{n_1-1} S[\frac{\rho}{c}(t_{j+1}-s_j)(e^{-ct_j}-e^{-cs_j})\ldots \qquad (8)$$

The total cost of supplier is given by

$$T_S\left(t_j,s_j,n_1\right)=n_1.Ss+Cs\sum_{j=0}^{n_1-1}\int_{t_j}^{s_{j+1}}(a-bp)\left(e^{s_{j+1}-\alpha t}-e^{(1-\alpha)t}\right)dt+\int_{s_j}^{t_j} e^{-ct}(t_j-s_j)$$

The primary goal is to determine the values of t_j and $s_j$that minimize the total variable cost (T_R) in stock control and inventory management, to achieve the lowest possible total cost in the inventory system, the essential conditions for minimizing the total cost are as follows,

$$\frac{\partial T_R(t_j,s_j,n_1)}{\partial t_j}=0 \ , \qquad j=1, 2, 3, \ldots.n$$

$$\frac{\partial T_R(t_j,s_j,n_1)}{\partial s_j}=0 \ , \qquad j=1, 2, 3, \ldots.n$$

$$\frac{\partial T_R(t_j,s_j,n_1)}{\partial t_j}=\sum_{j=0}^{n_1-1} H.(a-bp)[-e^{(s_{j+1}-\alpha t_j)}+e^{(1-\alpha)t_j}]+$$

$$\sum_{j=0}^{n_1-1} W_h(a-bp)\left[-\alpha e^{(s_{j+1}-\alpha t_j)}+(1-\alpha)e^{(1-\alpha)t_j}\right]+\sum_{j=0}^{n_1-1} S.\rho(t_{j+1}-s_j)e^{-at_j} \qquad \ldots (9)$$

$$\frac{\partial T_R(t_j,s_j,n_1)}{\partial s_j}=S.\sum_{j=0}^{n_1-1}\frac{\rho}{c}\left[e^{-ct_j}-e^{-cs_j}+c\left(t_{j+1}-s_j\right)e^{-s_j}\right] \qquad \ldots (10)$$

The total cost's Hessian matrix must be positive definite for a fixed n in order for the total cost to be least (i.e. $\nabla^2 TC$).

$\nabla T_R\left(t_j,s_j,n_1\right)=$

$$\begin{bmatrix}
\frac{\partial^2 T_R\left(t_j,s_j,n_1\right)}{\partial t_1{}^2} & \frac{\partial^2 T_R\left(t_j,s_j,n_1\right)}{\partial t_1\partial s_1} & 0 & 0 & \cdots & \cdots & 0 & 0 & 0 \\
\frac{\partial^2 T_R\left(t_j,s_j,n_1\right)}{\partial s_2\partial t_1} & \frac{\partial^2 T_R\left(t_j,s_j,n_1\right)}{\partial s_1{}^2} & \frac{\partial^2 T_R\left(t_j,s_j,n_1\right)}{\partial s_1\partial t_2} & 0 & \cdots & \cdots & 0 & 0 & 0 \\
0 & \frac{\partial^2 T_R\left(t_j,s_j,n_1\right)}{\partial t_2\partial s_1} & \frac{\partial^2 T_R\left(t_j,s_j,n_1\right)}{\partial t_2{}^2} & \frac{\partial^2 T_R\left(t_j,s_j,n_1\right)}{\partial t_2\partial s_2} & \cdots & \cdots & 0 & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & 0 & 0 & \cdots & \cdots & \frac{\partial^2 T_R\left(t_j,s_j,n_1\right)}{\partial t_{n_1-1}\partial s_{n_1-1}} & \frac{\partial^2 T_R\left(t_j,s_j,n_1\right)}{\partial s_{n_1-1}{}^2} & \frac{\partial^2 T_R\left(t_j,s_j,n_1\right)}{\partial s_{n_1-1}\partial t_{n_1}} \\
0 & 0 & 0 & 0 & \cdots & \cdots & 0 & \frac{\partial^2 T_R\left(t_j,s_j,n_1\right)}{\partial t_{n_1}\partial s_{n_1-1}} & \frac{\partial^2 T_R\left(t_j,s_j,n_1\right)}{\partial t_{n_1}{}^2}
\end{bmatrix}$$

4.3. Numerical illustration

A numerical example is presented with particular values for a=0.65, b=0.01, c=0.35, rho(ρ)=0.26, e=2.7, Wh=0.3, H=0.1, p=1.5, Sr=2.6748, s_1=0, S=4 in appropriate units. To obtain the solutions of Eq. (9) and Eq. (10), we utilized Mathematica, version 12, that is the computational tool implemented in order to compute the results and analysis for inventory model with backordering. The numerical example used in this study is inspired by parameter ranges and assumptions commonly found in existing literature on inventory models with backordering and price-sensitive demand. These parameters are chosen to reflect realistic supply chain conditions and to demonstrate the applicability of the proposed model under practical settings. The example is structured to validate the performance of the model under dynamic pricing, demand variability, and exponential backordering cost structure. While synthetic in nature, the chosen data set allows for a comprehensive sensitivity analysis and facilitates comparison with prior models.

4.4. Theorems

Theorem 1. First-Order Optimality Condition

Let f(x) be a differentiable function on R^n Then, necessarily, for $x^* \in \mathrm{R}^n$ to be a local minimum of f(x), the gradient of f(x) must be zero at x^*that is

$$\nabla f(x^*)=0$$

This condition is now known as first-order optimality.

Proof: The function f(x) is a local minimum at x^*, so no direction d satisfies:

$f(x^*+d) < f(x^*)$. From the first order Taylor expansion around x^*, we have

$$f\left(x^*+d\right) \approx f\left(x^*\right) + \nabla f(x^*)^T d$$

For that to be true for all directions d, it must be that $\nabla f(x^*)=0$.

Theorem 2 Second-order optimality condition

Let f(x) twice differentiable on R^nThe point $x^*$is a local minimum of f(x) if

$\nabla f(x^*)=0$ (first-order condition), and

The Hessian matrix $H(f)(x^*)$ is positive semi-definite that is, $H(f)(x^*) \geq 0$.

Proof: The second-order Taylor expansion of f(x) about x^* is

$$f(x^* d) \approx f\left(x^*\right)+\nabla f\left(x^*\right)^T d+\frac{1}{2} d^T H(f)\left(x^*\right) d$$

Because $\nabla f(x^*)=0$ (by the first-order condition), this gives

$$f(x^* d) \approx f\left(x^*\right)+\frac{1}{2} d^T H(f)\left(x^*\right) d$$

The inequality $f(x^*+d) \geq f(x^*)$ for all directions dd holds only if $H(f)(x^*)$ is positive semi-definite.

Theorem 3. KKT Conditions Karush-Kuhn-Tucker: The application of the Karush-Kuhn-Tucker (KKT) conditions in this research is justified by the nature of the optimization problem, which involves non-linear objective functions with inequality constraints related to inventory levels, replenishment periods, and backordering quantities. KKT conditions provide a well-established analytical framework to handle such constrained non-linear optimization problems, ensuring optimality under convexity and differentiability assumptions. While newer heuristic or metaheuristic methods (such as genetic algorithms or particle swarm optimization) are available, the use of KKT is appropriate here due to the analytical tractability of the model, which allows for closed-form expressions and clearer interpretation of the constraints and sensitivity of parameters. Furthermore, the combination of KKT conditions with symbolic computation in MATHEMATICA ensures robust and efficient computation, making it suitable for theoretical validation and pedagogical clarity. For the nonlinear programming problem:

$$\min f(x) \text{ subject to } \quad g_i(x) \leq 0, h_j(x)=0,$$

Karush-Kuhn-Tucker Conditions Suppose that f, g_i, h_j are differentiable. Karush-Kuhn-Tucker conditions give necessary optimality conditions. Let x^*be a local minimum. Then there exist multipliers $\lambda_i \geq 0$ and μ_j such that

$$\nabla f\left(x^*\right) + \sum i\lambda i\nabla gi\left(x^*\right) + \sum j\mu j\nabla hj\left(x^*\right) = 0$$

$$\lambda_i g_i(x^*) = 0 \quad \text{for all i, (complementaryslackness),}$$

$$g_i\left(x^*\right) \leq 0, \; h_j\left(x^*\right)= 0 \text{ for all i, j.}$$

$$\lambda_i \geq 0 \text{ for all i.}$$

5. Resulting tables and figures

Table 1. Replenishment time t_j

↓***a*** → t_j	t_0	t_1	t_2	t_3
0.1	0.01	0.122146		
0.2	0.01	2.37729	3.33812	
0.4	0.01	1.71775	3.33812	4
0.6	0.01	1.71775	3.33812	4

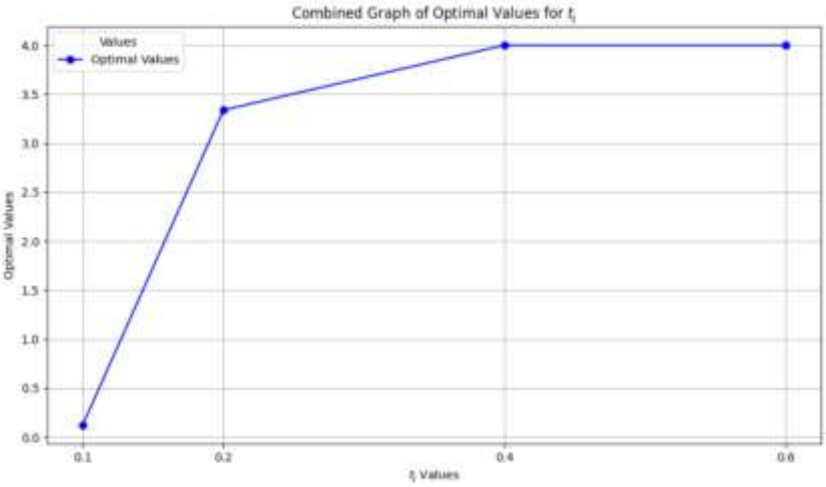


Figure 1. Replenishment time t_j

Table 2. Increasing replenishment time s_j

↓ *a* →s_j	s_0	s_1	s_2	s_3
0.1	0	3.99881		
0.2	0	2.37743	3.9978	
0.3	0	1.71775	3.33812	4
0.4	0	1.71775	3.33812	4

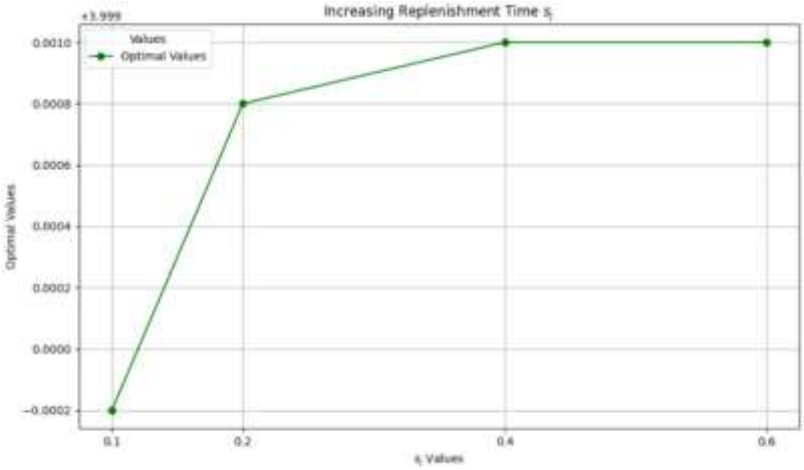


Figure 2. Increasing order of replenishment time s_j

Table 3. Total cost for the retailer for different replenishment cycle

↓ a	T_R	T_S	Q_{nt}
0.1	4.13456	2.67411	2.67255
0.2	6.33814	2.67388	6.8361
0.4	8.53743	5.33857	7.86121
0.5	9.40242	5.33598	10.0195
0.6	10.2558	5.33346	12.1581

Table 4. Total cost for retailer, supplier and quantity for optimal values

→n	**1**	**2**	**3**	**4**	**5**	**6**
0.1	**4.13456**	5.85722	9.00915	12.9165	18.3473	27.1847
0.2	**6.33814**	6.76708	10.272	15.9013	25.1825	42.6298
0.4	9.44631	**8.53743**	12.7736	21.8019	38.6834	73.1256
0.5	10.5904	**9.40242**	14.0225	24.7468	45.4208	88.343
0.6	11.5592	**10.2558**	15.2711	27.6907	52.1556	103.554

Table 5. Days table for t_j

148.156	313.488	609.206	904.924
19.7869	433.88	609.206	904.924
1.825	313.488	609.206	904.924
1.825	313.488	609.206	904.924

Table 6. Days table for s_j

0	729.783	609.206	904.924
0	433.88	729.598	904.924
0	313.488	609.206	904.924
0	313.488	609.206	904.924

6. Sensitivity analysis

The sensitivity analysis elucidates the impact of changes in the crucial parameters on total costs for the retailer T_R, supplier T_S, and the total order quantity Q_{nt}. A 20% increase in parameter a causes T_R to shift up to 9.40242 from 8.53743 and T_S shifts down to its slightly decreased value from 5.33857 to 5.33598. The total order quantity Q_{nt} shifts up to 10.0195 from 7.86121. On the other hand, with the fall of 20% in a, T_R falls to 6.33814 and T_S falls down to 2.67388, and Q_{nt} also falls down to 6.8361. This shows that it is moderately sensitive to fluctuations in the cost and the order quantity of a. Changes have very minimal effects on parameter b. T_R varies within the range of 7.05339 to 7.04076 and T_S varies very slightly as it ranges from 0.00072145 to 0.000693299 whilst the total order quantity decreases slightly by the range of 4.0448 to 4.00439. This also shows that b has lesser effects on costs and order quantity Q_{nt}.

This shows that parameter c is fairly sensitive. In fact, a 20% drop resulted in T_R sharply increasing from 6.87698 to 11.3531, T_S from 0.00101634 to 0.00129504, and quantity Q_{nt} from 3.7784 to 5.36078. α also turns out to be highly sensitive. Indeed, a 20% increase in it leads to a sharp increase in T_R from 5.37491 to 29.9734, and T_S from 0.002066 to 0.0131941, while quantity Q_{nt} decreases from 3.03674 to 2. 44819.Therefore, c and α are actually important parameters, which largely dictate total costs, apart from the order quantities especially for the retailer. For graphical visualizations a Machine-Learning Tool-Python, with the help of it the heatmap are made. Figure 3 gives the visualization of the sensitivity of retailer's total cost, Figure 4 gives the graphical visualization of sensitivity of supplier's total cost and in Fig 5 gives the graphical visualization of optimal order quantity.

Table 7. Sensitivity analysis of all the parameters

Parameters	%Changes	Optimal Replenishment cycle	Total order Quantity Q_{nt}	Total cost of Retailer T_R	Total cost of supplier T_S
a	+20 +10 0 -10 -20	2	10.0195	9.40242	5.33598
		2	12.1581	10.2558	5.33346
		2	7.86121	8.53743	5.33857
		1	2.67255	4.13456	2.67411
		1	6.8361	6.33814	2.67388
b	+20 +10 0 -10 -20	1	4.00439	7.04076	0.000693299
		1	3.96288	7.02916	0.000662191
		1	4.0448	7.05339	0.00072145
		1	4.16037	7.09689	0.000790488
		1	4.1227	7.08151	0.000769832
c	+20 +10 0 -10 -20	1	3.32059	5.95992	0.00114326
		1	2.92617	5.29343	0.00114757
		1	3.7784	6.87698	0.00101634
		1	5.36078	11.3531	0.00129504
		1	4.82145	9.57865	0.000356569
α	+20 +10 0 -10 -20	1	2.44819	4.21495	0.00286675
		1	2.20027	4.04957	0.00278454
		1	3.03674	5.37491	0.002066
		1	15.6442	29.9734	0.0131941
		1	8.74641	16.4004	0.00507976
W_h	+20 +10 0 -10 -20	1	3.44331	6.64752	0.00139882
		1	3.22745	6.49588	0.00165342
		1	3.7784	6.87698	0.00101634
		1	10.5587	9.24053	0.00280211
		1	5.72052	7.96899	0.000725277
H	+20 +10 0 -10 -20	1	3.92663	7.28774	0.000701578
		1	4.08092	7.71854	0.00038045
		1	3.7784	6.87698	0.00101634
		1	3.36164	5.73868	0.00192955
		1	3.49676	6.1048	0.00162995

Parameters	%Changes	Optimal Replenishment cycle	Total order Quantity Q_{nt}	Total cost of Retailer T_R	Total cost of supplier T_S
Rho	+20	1	4.72833	8.0726	0.00100185
	+10	1	5.35694	9.13329	0.00114567
	0	1	4.0448	7.05339	0.00072145
	-10	1	1.69676	4.42553	0.0010419
	-20	1	2.53281	5.2142	0.00027822
S	+20	1	4.0607	7.84175	0.00119037
	+10	1	4.35968	8.85483	0.00129419
	0	1	3.7784	6.87698	0.00101634
	-10	1	2.96604	4.39282	0.00233261
	-20	1	3.22041	5.14019	0.000300246

Table 7. (*Continued*)

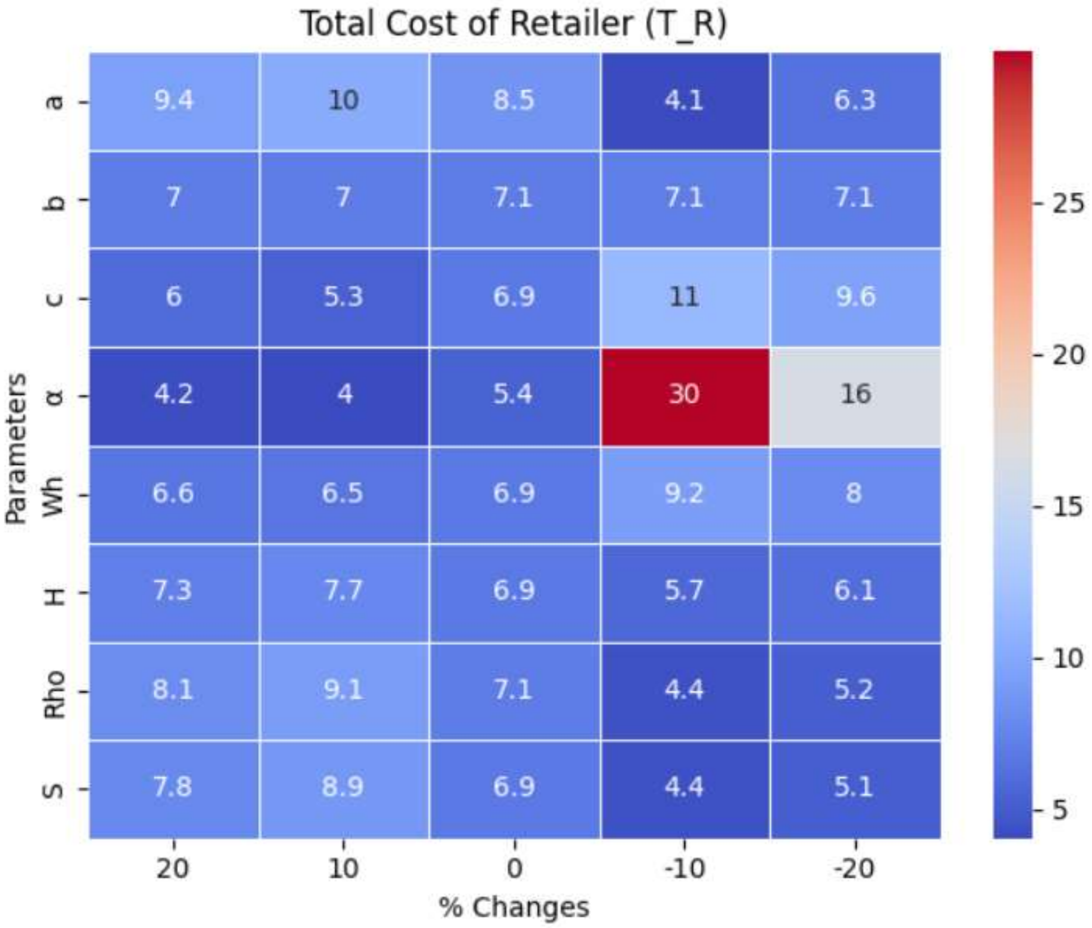
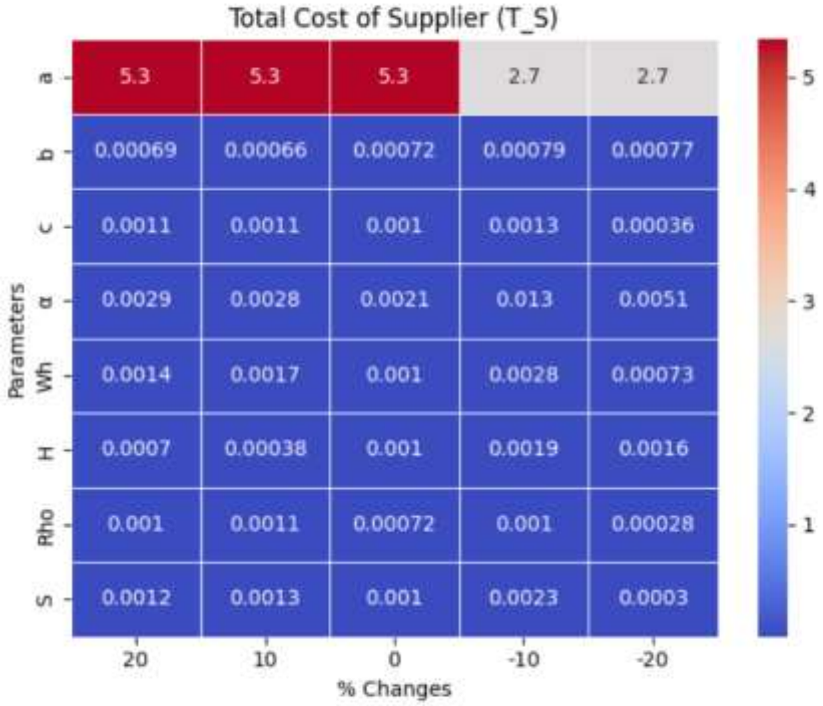
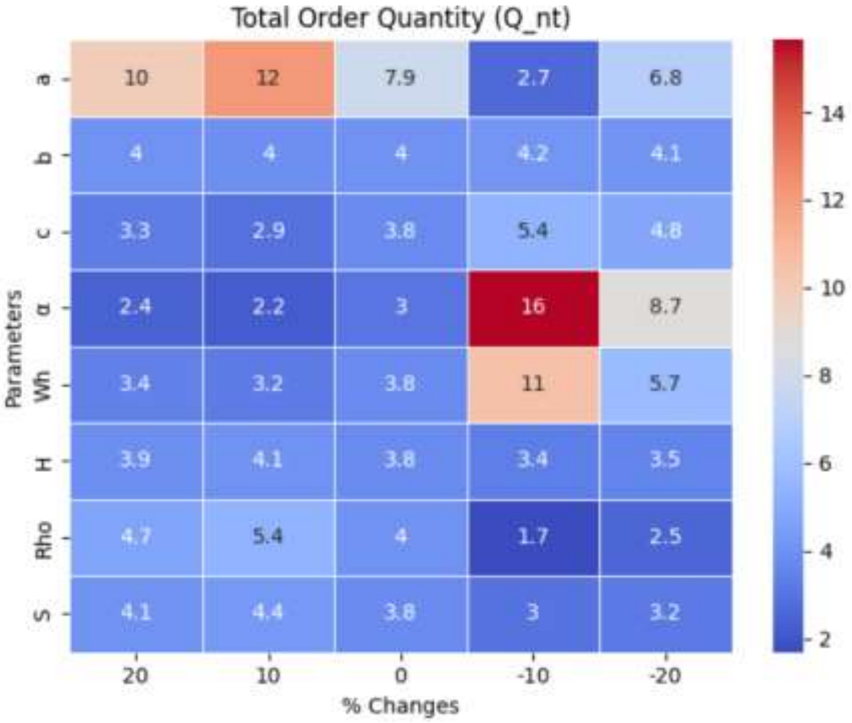


Figure 3. Heatmap of retailer's cost: Tracking the ripple effect of parameter changes


Total Cost of Supplier (T_S)

Parameters	20	10	0	-10	-20
a	5.3	5.3	5.3	2.7	2.7
b	0.00069	0.00066	0.00072	0.00079	0.00077
c	0.0011	0.0011	0.001	0.0013	0.00036
α	0.0029	0.0028	0.0021	0.013	0.0051
Wh	0.0014	0.0017	0.001	0.0028	0.00073
H	0.0007	0.00038	0.001	0.0019	0.0016
Rho	0.001	0.0011	0.00072	0.001	0.00028
S	0.0012	0.0013	0.001	0.0023	0.0003

Figure 4. Heatmap of supplier's cost: Tracking the ripple effect of parameter changes


Total Order Quantity (Q_nt)

Parameters	20	10	0	-10	-20
a	10	12	7.9	2.7	6.8
b	4	4	4	4.2	4.1
c	3.3	2.9	3.8	5.4	4.8
α	2.4	2.2	3	16	8.7
Wh	3.4	3.2	3.8	11	5.7
H	3.9	4.1	3.8	3.4	3.5
Rho	4.7	5.4	4	1.7	2.5
S	4.1	4.4	3.8	3	3.2

Figure 5. Total order quantity trends with parameter variations

7. Managerial suggestions

To benefit from back ordering to the fullest and to minimize total costs, the managers should initiate strategic moves, starting with optimum inventory levels based on accurate demand forecasting. Advanced techniques of forecasting to determine fluctuations in demand and when to allow back orders would indeed lead to both fulfilling customer orders on time and reducing excess holding costs. Various suppliers could be created to allow them to negotiate good terms for the back-ordered items, thus ensuring that the supplier stabilizes the cost during peak demand T_S, while such flexibility may enable managers to avoid misunderstandings or overreacting3 to changes in the marketplace. Again, providing discounts, loyalty points, or priority filling as an incentive to encourage customers to take backordered items can improve the satisfaction level of customers and optimize cash flows. This would enable customers to wait for the products and therefore minimize the loss of sales. The other thing of huge importance is conducting periodic backorders analysis. Analysis would permit managers to know the after-effects of back-ordering on efficiency in operations as well as customer satisfaction. This would make the necessary adjustments to the responsible strategies. With such controls, organizations can rightly monitor back ordering, lower their overall cost, and potentially increase profit without losing the relationship with the customer.

8. Conclusion

In this paper, an exhaustive exponential complete backordering inventory model is developed for countering shortages at every cycle of a finite horizon with both linear trends and price-sensitive demands. An exhaustive, exponential complete backordering model will offer some significant improvement in the inventory management by considering exponential backordering, giving better applicability towards the real-world demand, where customers' demand is sensitive to the pricing and supply lags. Mathematica's mathematical verification, combined with computation support from the tools based on Python, makes this model even more versatile in giving accurate numerical solutions and intelligent graphical analysis.

Results of the numerical example will show that the model is optimizing the total cost while improving coordination between the suppliers and retailers. Results of the sensitivity analysis will provide an intuitive sense that total costs are sensitive to the back ordering rate, c, and the inventory dependent coefficient, α, therefore, the importance of these parameters in the strategic decision-making for the inventory manager can be realized. This implies that the back ordering policies can be optimized by reducing stockouts in order to maximize customer satisfaction by a cost-effective supply chain.

Summarily, the developed model gives practitioners and decision-makers a critical tool for looking forward to improving the mechanisms of controlling inventory. Effective management of back orders under fluctuating demand and pricing conditions will ensure not only cost optimization but also improvements in efficiency in operations. This model may be extended into multi-echelon supply chains, with consideration for stochastic demand patterns and implications of dynamic pricing strategies.

Conflict of interest

The authors declare no conflict of interest.

References

1. Steven Nahmias and S. Nahmias, “Production and Operations Analysis, 6th Edition,” Jan. 2014.
2. David Simchi-Levi and D. Simchi-Levi, “Designing And Managing The Supply Chain,” May 2005.
3. Stephen C. Graves and S. C. Graves, “Reflections on the Evolution of Operations Management,” Manag. Sci., 2020, doi: 10.2139/ssrn.3636391.
4. Snyder Shen, “Applications of Supply Chain Theory,” Fundam. Supply Chain Theory, 2019, doi: 10.1002/9781119584445.ch16.
5. Arzum Akkaş, A. Akkas, Dorothée Honhon, and D. Honhon, “Shipment Policies for Products with Fixed Shelf Lives: Impact on Profits and Waste,” Manuf. Serv. Oper. Manag., Jul. 2020, doi: 10.2139/ssrn.3247290.
6. Gérard P. Cachon, G. P. Cachon, Christian Terwiesch, and C. Terwiesch, “Matching Supply with Demand,” Jan. 2000.
7. S.C. Graves, S.C. Graves, A.H.G. Rinnooy Kan, A.H.G. Rinnooy Kan, Paul Zipkin, and P.H. Zipkin, “Preface,” Handb. Oper. Res. Manag. Sci., 1993, doi: 10.1016/s0927-0507(05)80180-x.

8. Marshall L. Fisher, M. L. Fisher, Ananth Raman, and A. Raman, “The New Science of Retailing: How Analytics are Transforming the Supply Chain and Improving Performance,” Jun. 2010.
9. Srinagesh Gavirneni, S. Gavirneni, Roman Kapuściński, R. Kapuscinski, Sridhar Tayur, and S. Tayur, “Value of Information in Capacitated Supply Chains,” Manag. Sci., vol. 45, no. 1, pp. 16–24, Jan. 1999, doi: 10.1287/mnsc.45.1.16.
10. Renuka S. Namwad, N. K. Mishra, and Ranu, “Trade Credit and Preservation Technologies: An Inventory Replenishment Model for a Sustainable Supply Chain,” J. Eur. Syst. Autom., 2023, doi: 10.18280/jesa.560613.
11. Sunil Chopra, S. Chopra, Peter Meindl, and P. Meindl, “Supply Chain Management: Strategy, Planning and Operations,” May 2003, doi: 10.1007/978-3-8349-9320-5_22.
12. David A. Wuttke et al., “Supply chain finance: Optimal introduction and adoption decisions,” Int. J. Prod. Econ., vol. 178, pp. 72–81, Aug. 2016, doi: 10.1016/j.ijpe.2016.05.003.
13. S. Kumar, “An Integral Inventory Model for Decaying Products with Price-Rate Dependent Demand, Exponential Decay, Shortages and Complete Backlogging,” vol. 40, no. 74, pp. 2148–2156, Mar. 2020.
14. Joseph Milner, J. Milner, Meir J. Rosenblatt, and M. J. Rosenblatt, “Flexible supply contracts for short life-cycle goods: The buyer’s perspective,” Nav. Res. Logist., vol. 49, no. 1, pp. 25–45, Feb. 2002, doi: 10.1002/nav.10002.
15. Vic Anand, Ramji Balakrishnan, and Srinagesh Gavirneni, “Capacity planning with limited information,” Prod. Oper. Manag., vol. 32, no. 9, pp. 2740–2757, Apr. 2023, doi: 10.1111/poms.14004.
16. Akshay R. Rao, A. R. Rao, Mark Bergen, M. Bergen, Scott T. Davis, and S. Davis, “How to fight price a war,” Harv. Bus. Rev., vol. 78, no. 2, pp. 107–116, Dec. 2000.
17. Kris Johnson Ferreira, K. J. Ferreira, Bin Hong Alex Lee, Bin Hong Alex Lee, David Simchi-Levi, and D. Simchi-Levi, “Analytics for an Online Retailer: Demand Forecasting and Price Optimization,” Manuf. Serv. Oper. Manag., vol. 18, no. 1, pp. 69–88, Feb. 2016, doi: 10.1287/msom.2015.0561.
18. Hau L. Lee, H. L. Lee, Vineet Padmanabhan, V. Padmanabhan, Seungjin Whang, and S. Whang, “The Bullwhip Effect in Supply Chains,” Sloan Manage. Rev., vol. 38, no. 3, pp. 93–102, Jan. 1997, doi: 10.1109/emr.2015.7123235.
19. Paul Zipkin and P. Zipkin, “Foundations of Inventory Management,” Jan. 2000.
20. Charles P. Schmidt, C. P. Schmidt, Steven Nahmias, and S. Nahmias, “S-1, S Policies for Perishable Inventory,” Manag. Sci., vol. 31, no. 6, pp. 719–728, Jun. 1985, doi: 10.1287/mnsc.31.6.719.
21. Awi Federgruen, A. Federgruen, Paul Zipkin, and P. Zipkin, “An Inventory Model with Limited Production Capacity and Uncertain Demands II. The Discounted-Cost Criterion,” Math. Oper. Res., vol. 11, no. 2, pp. 208–215, May 1986, doi: 10.1287/moor.11.2.208.
22. A. J. Clark, A. J. Clark, Herbert E. Scarf, and H. E. Scarf, “Optimal Policies for a Multi-Echelon Inventory Problem,” Manag. Sci., vol. 50, no. 12, pp. 1782–1790, Jul. 1960, doi: 10.1287/mnsc.1040.0265.
23. Hau L. Lee, “Managing supply chain inventory: Pitfalls and opportunities,” 1992.
24. Evan L. Porteus and E. L. Porteus, “Investing in Reduced Setups in the EOQ Model,” Manag. Sci., vol. 31, no. 8, pp. 998–1010, Aug. 1985, doi: 10.1287/mnsc.31.8.998.
25. John A. Buzacott et al., “Safety stock versus safety time in MRP controlled production systems,” Manag. Sci., vol. 40, no. 12, pp. 1678–1689, Dec. 1994, doi: 10.1287/mnsc.40.12.1678.
26. Guillermo Gallego, G. Gallego, Garrett van Ryzin, and G. van Ryzin, “Optimal dynamic pricing of inventories with stochastic demand over finite horizons,” Manag. Sci., vol. 40, no. 8, pp. 999–1020, Aug. 1994, doi: 10.1287/mnsc.40.8.999.
27. Marshall L. Fisher, M. L. Fisher, Ananth Raman, and A. Raman, “Reducing the Cost of Demand Uncertainty Through Accurate Response to Early Sales,” Oper. Res., vol. 44, no. 1, pp. 87–99, Feb. 1996, doi: 10.1287/opre.44.1.87.
28. Michael Ketzenberg, M. Ketzenberg, Mark Ferguson, and M. Ferguson, “Managing Slow-Moving Perishables in the Grocery Industry,” Prod. Oper. Manag., vol. 17, no. 5, pp. 513–521, Sep. 2008, doi: 10.3401/poms.1080.0052.
29. Jain, P., & Mishra, N. K. (2024). Enhancing Supply Chain Efficiency with Blockchain: Addressing Information Sensitivity for Increased Manufacturer Profitability. IFAC-PapersOnLine, 58(19), 688–693. https://doi.org/10.1016/j.ifacol.2024.09.220
30. Mishra, N. K., Jain, P., & Ranu. (2024). Blockchain-Enhanced Inventory Management in Decentralized Supply Chains for Finite Planning Horizons. Journal Européen Des Systèmes Automatisés, 57(1), 263–272. https://doi.org/10.18280/jesa.570125