
Original Research Article

Performance analysis of non-orthogonal-multiple-access within massive MIMO systems

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Abstract: NOMA and massive MIMO have both been proposed for achieving SE and EE in 5G wireless networks. However, some studies have considered the combination of both technologies in a single system. Accordingly, this paper investigates the performance of NOMA within massive MIMO for the first time. The analysis adopts a random matrix theory method. The method and considers both the case where the users share the same frequency for transmission purposes and the case where each user can use different frequencies for its transmissions. A new SINR performance measure is proposed. Closed-form expressions for the BER, outage probability and outage capacity are additionally derived.

Keywords: Cellular networks; NOMA; Massive MIMO

1. Introduction

Within the background of 5G, NOMA^[1], small cell^[2], massive MIMO^[3] and HetNets^[4] have all been proposed for improving the SE and EE of 5G architectures, the optimum choice of antenna and Radio Chain, the derivation of closed-form expressions for the channel capacity, the development of efficient encoding techniques, and so on. The authors in^[1] increase the performance of NOMA over block-fading channels by utilizing retransmissions to improve the diversity gain and SIC to enhance the SIR. NTT DoCoMo Inc.^[5,6] investigated the performance of NOMA within a shared spectrum and compared it with that of conventional OFDM technology. Niu et al.^[7] proposed a technique referred to as cell zooming for adjusting the cell size in massive MIMO architectures adaptively in accordance with changes in the traffic load, user requirements and channel conditions. It was shown that the proposed approach yielded a significant reduction in the power consumption of cellular networks. Zhou et al.^[8] presented an power-efficient antenna selection and energy allocation algorithm for maximizing the EE in large-scale MIMO. However, in deriving the algorithm, the effect of the BS coverage area was neglected. Accordingly, the authors in^[9] proposed a method designated as CPZ for improving the EE in massive MIMO systems by zooming the BS coverage area in or out to maintain the coverage area or minimize the power consumption, respectively.

Usually, previous studies focus on either NOMA or massive MIMO. That is, the simultaneous deployment of both technologies in a single system has not been addressed. However, combining NOMA with massive MIMO provides the potential to improve both the SE (through the use of NOMA) and the EE (through the use of massive MIMO). Accordingly, the present research utilizes a random matrix theory approach to analyze the performance of NOMA within massive MIMO for the first time. The analysis considers two different resource allocation scenarios, namely frequency sharing (in which multiple users share the same frequency for transmission purposes) and frequency multiplexing (in which each user is permitted to use different frequencies to perform its transmissions). Two new SINR performance measures are proposed to quantify the SINR within the same frequency and within discriminated frequencies, respectively.” OK?) Closed-form expressions for the

BER, outage probability and outage capacity are additionally derived.

2. Proposed system model

Theoretically, massive MIMO and small-cell HetNets can be deployed in both indoor and outdoor situations. However, massive MIMO is generally preferred for outdoor situations. since it supports large-scale spatial multiplexing and hence improves the EE performance. By contrast, small-cell HetNets are suitable for indoor situations.

The research considers the case of a massive MIMO system with N antennas serving a lot of K UEs. In traditional massive MIMO systems, each BS independently manages its own resources. However, in developing the present model, it is assumed that the system employs CoMP and the resources of all the BSs are remotely scheduled and coordinated in the cloud with the assistance of a CRAN^[10].

The channels are assumed to experience independent fast fading and slow fading which can also be expressed by the geometric attenuation and log-normal shadow fading. Thus, the channel coefficient, $h_{m,k}$, is taken as $h_{m,k} = a_{m,k} \sqrt{\beta_{m,k}}$, where $a_{m,k}$ is the fast fading coefficient from the m th to the k th UE and $\sqrt{\beta_{m,k}}$ is the geometric attenuation and shadowing fading coefficient, which can be further expressed as $\beta_{m,k} = \frac{1}{1 + d_{m,k}^\alpha}$, where $d_{m,k}$ denotes the distance from the antenna to the UE and α is the path loss factor. This is the general Rayleigh channel for MIMO systems, in which the channel matrix obeys $H \sim CN(0, \lambda_k I_k)$, where λ_k is the k th largest eigenvalue of HH^H .

Usually, it is assumed that each BS is located at the center of a circular coverage area with radius \mathfrak{R}_d and the antenna arrays are equipped with omnidirectional directions. That is, the antenna array can be regarded as a cylindrical structure with orderly aligned antennas with same distances by the column and row directions. It is further assumed that the UEs are uniformly distributed within the coverage area and the transmitted signal is given by

$$S = \sqrt{P} Hx, \tag{1}$$

where \sqrt{p} is the allocated power for the UE transmission. Taking W as the precoding matrix, and adopting ZFBF, the matrix can be further written as

$$S = \sqrt{p} H^H (HH^H)^{-1} x. \tag{2}$$

Adding the precoding information, the transmitted signal can be written as

$$S = \sqrt{p} H^H (HH^H)^{-1} x. \tag{3}$$

Thus, the received signal of the k th UE is given by

$$\begin{aligned} y_k &= HS_k + n_k = HWx_k + n_k \\ &= \sqrt{p} HH^H (HH^H)^{-1} x_k + n_k \\ &= \sqrt{P_k} \|H_k\|^2 x_k + \sqrt{P} \sum_{i=1, i \neq k}^K H_i^H H_i x_i + n_k, \end{aligned} \tag{4}$$

where $E[H_{i,j}] = 0, E[H_{i,j}^2] = 0, E[|H_{i,j}|^2] = 1, E[nn^H] = \sigma^2 I$.

3. Performance analysis of massive MIMO system with NOMA

In a typical NOMA system, each frequency bands are shared by multiple users, and each user is allocated

a different energy to perform its transmissions. However, this research considers the additional multiplexing cases, in which the users are permitted to use different frequencies to perform their transmissions. Previous NOMA studies assume that following SIC, all of the data can be successfully decoded. However, in practical implementations, this is rarely the case; particularly in lower SINR channel, in which the noise may outweigh the discriminated gap between the different powers, thereby rendering the decoding operation intractable.

In a massive MIMO system with NOMA encoding, the received signal can be expressed as

$$\begin{aligned}
 y_1 &= \overbrace{[h_{1,1}(p_{1,1} + p_{2,1} \dots + p_{k,1})x_{1,1} + n_{1,1}]}^{\text{within the same frequency}} + \overbrace{[h_{1,2}(p_{1,2} + p_{2,2} \dots + p_{k,2})x_{1,2} + n_{1,2}] \dots + [h_{1,n}(p_{1,n} + p_{2,n} \dots + p_{k,n})x_{1,n} + n_{1,n}]}^{\text{within the discriminated frequency}} \\
 y_2 &= [h_{2,1}(p_{1,1} + p_{2,1} \dots + p_{k,1})x_{2,1} + n_{2,1}] + [h_{2,2}(p_{1,2} + p_{2,2} \dots + p_{k,2})x_{2,2} + n_{2,2}] \dots + [h_{2,n}(p_{1,n} + p_{2,n} \dots + p_{k,n})x_{2,n} + n_{2,n}] \\
 &\vdots \\
 &\vdots \\
 y_k &= [h_{k,1}(p_{1,1} + p_{2,1} \dots + p_{k,1})x_{k,1} + n_{k,1}] + [h_{k,2}(p_{1,2} + p_{2,2} \dots + p_{k,2})x_{k,2} + n_{k,2}] \dots + [h_{k,n}(p_{1,n} + p_{2,n} \dots + p_{k,n})x_{k,n} + n_{k,n}]
 \end{aligned} \tag{5}$$

In other words, within the same spectrum bandwidth, the received signal of the k th UE is given by

$$y'_k = \sum_{i=1}^K h_{i,1} p_i, \tag{6}$$

while the received signal from the neighboring spectrum bandwidth is given as

$$y''_k = \sum_{i=1}^K \sum_{j=1}^M h_{i,j} p_{i,j}. \tag{7}$$

Thus, the sum received signals at the k th UE can be expressed as

$$y_k = \sum_{i=1}^K h_{i,1} p_i + \sum_{i=1}^K \sum_{j=1}^M h_{i,j} p_{i,j}. \tag{8}$$

In analyzing the SINR at the k th UE, it is necessary to consider both the SINR within the same frequency and the SINR associated with the IFI caused by neighboring frequencies. In other words, the SINR can be formulated as follows:

$$SINR_{k,1} = \frac{\overbrace{\rho |h_{1,k}|^2 \alpha_k}^{\text{within the same frequency}}}{\rho |h_{1,k}|^2 \sum_{i=1, i \neq k}^K \alpha_i + 1}, \tag{9}$$

$$SINR_{k,2} = \frac{\overbrace{\sum_{i=2}^K \rho |h_{i,k}|^2 \alpha_{i,k}}^{\text{within the discriminated frequency}}}{\sum_{i=1, i \neq k}^K \rho |h_{i,k}|^2 \alpha_{i,k} + 1}. \tag{10}$$

The authors in^[11] developed a universal marginal CDF expression for the channel matrix distribution in the double-scattering channels of MIMO multichannel beamforming systems. The authors in^[12] derived a similar expression for MIMO systems with Rayleigh or Rician channels. Since the SINR considered in the present model comprises two components (namely within the same frequency and within discriminated frequencies), a similar method to that used in^[11,12] can be used to determine the BER and outage performance within the same channel with ordered power allocation. The aim of ZFBF is to maximize the SINR within the cellular communication system. The instantaneous output SINR of user k can thus be expressed by the following channel matrix:

$$\gamma_k = \overline{\gamma_k} \omega^H H^H H \omega = \frac{P_k}{n_k} \omega^H H^H H \omega, \tag{11}$$

where $\overline{\gamma_k} = p_k / n_k$ is the SNR of the k th user.

As in [13], the BER can be obtained as

$$BER_a = \int_0^\infty \left(\int_{\sqrt{x}}^\infty \frac{1}{\sqrt{2\pi}} \exp^{-\frac{t^2}{2}} dt \right) dF_{SINR}(x) \tag{12}$$

and the average SER is given by

$$\begin{aligned} \overline{SER} &= E[SER(\xi \overline{SNR})] \\ &= \frac{a\sqrt{b}}{2\sqrt{pi}} \int_0^\infty x^{\frac{1}{2}} \exp^{-bx} F_\xi\left(\frac{x}{SNR}\right) dx, \end{aligned} \tag{13}$$

where $F_\xi(\cdot)$ is the CDF of the random variable ξ . In accordance with^[12], the joint PDF of the ordered eigenvalues, $\lambda_1 \leq \lambda_2 \dots \leq \lambda_n \leq 0$, within a Rayleigh fading channel is given as

$$f_\lambda(x) = \frac{1}{\prod_{i=1}^K (N-i)!(K-i)!} |\Lambda(\lambda)|^2 \prod_{i=1}^n \exp^{-\lambda_i} \lambda_i^{K-i}, \tag{14}$$

where $\Lambda(\cdot)$ is a Vandermonde matrix. Given a knowledge of the marginal CDF of the smallest nonzero eigenvalue λ_n , the outage probability can be obtained as

$$P = P_r(\rho_k \leq \bar{\rho}) = F_{\lambda_k} \left(\frac{\bar{\rho}}{\phi_k \overline{SNR}} \right), \tag{15}$$

where ϕ is defined as $\sum_{i=1}^k \phi_i = 1$. The channel capacity can be expressed in terms of the channel capacity theorem as $C = \log_2(1 + SINR)$. Thus, the achievable outage capacity complementary cumulative probability can be given as

$$\begin{aligned} P_r(C \leq C_{th}) &= 1 - P_r(SINR < 2^{C_{th}-1}) \\ &= 1 - P_r(\overline{\gamma}_k \omega^H H^H H \omega < 2^{C_{th}-1}) \\ &= 1 - P_r\left(\frac{P_k}{n_k} \omega^H H^H H \omega < 2^{C_{th}-1}\right) \end{aligned}, \tag{16}$$

where C_{th} is the threshold of the outage capacity.

4. Conclusion

This study has, for the first time, investigated the performance of NOMA in massive MIMO. In particular, expressions have been derived for the SINR at the k-th UE within the same channel and within discriminated frequencies, respectively. In addition, closed-form expressions have been presented for the BER, the outage probability and the outage capacity. One can convince that while combining the two techniques together, both the SE and EE performance can be enhanced in cellular communication networks. This will be further studied in our next step. Future research will investigate the potential for improving the SE and EE performance of cellular communication networks by integrating the NOMA and MIMO technologies.

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