Original Research Article

Analysis of channel capacity for LTE downlink multiuser MIMO systems

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Abstract: Abstract—The average channel capacity for 3GPP LTE down-link multiuser Multiple Input Multiple Output (MIMO) systems is analyzed in this paper. A packet scheduler is used to exploit the available multiuser diversity in all the three physical domains (i.e., space, time and frequency). A mathematical model is established to derive the channel capacity of multiuser MIMO systems with the frequency domain packet scheduler (FDPS). This work provides a theoretical reference for the future version of the LTE standard and a useful source of information for the practical implementation of the LTE systems.

Keywords: Channel capacity; LTE, OFDMA; Multiuser MIMO

1. Introduction

LTE standard is considered as one of key candidates for the next generation wireless communications. LTE air interface is optimized for the higher spectral efficiency and short transmit latencies using advanced modulation, link adaptation. More importantly, MIMO techniques in combination with Orthogonal Frequency Division Multiple Access (OFDMA) have been adopted by the LTE standard[1].

Most existing work on linear precoding focuses on the design of the transmitter precoding matrix, e.g.,[2,3]. In[4,5], the interaction between packet scheduling and array antenna techniques is studied based on a system level simulation model. The interactions between multiuser diversity and spatial diversity is investigated analytically in[6], with the focus on space time block coding. In this paper, we conduct a theoretical analysis for the average channel capacity in multiuser MIMO systems with FDPS, which has not been covered in the existing literature so far. Our analysis reveals that the outage probability for systems using single-user (SU) MIMO scheme is generally larger than the one with multi-user (MU) MIMO scheme, and linear precoding can improve the average channel capacity for the investigated MIMO systems.

2. Signal model

The basic scheduling unit in LTE is the Physical Resource Block (PRB), which consists of a number of consecutive OFDM sub-carriers reserved during the transmission of a fixed number of OFDM symbols. Two spatial division multiplexing (SDM) schemes have been considered in LTE standard, i.e., SU-MIMO and MU-MIMO[1]. The Frequency Domain (FD) scheduling algorithm considered in this work is the FD Proportional Fair (PF)[7,8] packet scheduling algorithm, which is being investigated under LTE.

The studied system has n_t transmit antennas at the Base Station (BS) and n_r^i receive antennas at the i th Mobile Station (MS), $i = 1,2,,...,K$ .Without loss of generality, we assume that all the MSs have equal numbers of antennas n_r, and define $M = \min(n_t, n_r)$ and $N = \max(n_t, n_r)$ .The scheduler in BS select at most n_t users per PRB from the K active users in the cell for data transmission. Denote by ζ_k the set of users scheduled on the k th PRB and $|\zeta_k| = n_t$. Without precoding, the received signal vector at the j th MS, $j \in \zeta_n$, can be modeled as $\mathbf{y}_{n,j} = \mathbf{Ç}_{n,j}\mathbf{x}_n + \mathbf{n}_{n,j}$, where $\mathbf{n}_{n,j} \in \mathbf{C}^{n_r \times 1}$ is a circularly symmetric complex Gaussian noise vector with zero mean and covariance matrix $N_0\mathbf{I} \in \mathbf{R}^{n_r \times n_r}$,i.e., $\mathbf{n}_{n,j} \sim CN(\mathbf{0}, N_0\mathbf{I})$; $\mathbf{H}_{n,j} \in \mathbf{C}^{n_r \times n_t}$ is the channel matrix between the BS and the j th MS at the n th PRB and $\mathbf{x}_n \in \mathbf{C}^{n_t \times 1}$ is the transmitted signal vector at the n th PRB, and the μ th element of $\mathbf{x}_n$ is the data symbol $x_{n,\mu}$ transmitted from the μ th MS, $\mu \in \zeta_n$, can be formed as

$$\mathbf{y}_{n,j} = \mathbf{H}_{n,j} \sum_{\mu \in \zeta_n} \mathbf{b}_{n,\mu} x_{n,\mu} + \mathbf{n}_{n,j} = \mathbf{H}_{n,j} \sum_{\mu \in \zeta_n} \mathbf{B}_{n,\mu} \phi_{n,\mu} + \mathbf{n}_{n,j} . \tag{1}$$

where $\mathbf{b}_{n,\mu} \in \mathbf{C}^{n_t \times 1}$ is the beamvector for the μ th MS user data on the n th PRB and $\mathbf{B}_{n,\mu} \in \mathbf{C}^{n_t \times n_t}$ is the precoding matrix with the μ th column of $\mathbf{B}_{n,\mu}$ equal to $b_{n,\mu}, \psi_{n,\mu} \in C^{n_t \times 1}$ is a column vector in which the μ th element equal to $x_{n,\mu}$ and the rest equal to zero.

For MU-MIMO SDM scheme, we assumer the MS only report quantized channel state information to the BS. The BS select the users with the same quantized channel state information for MIMO transmission. In this case, the precoding matrices for the selected users will be the same, therefore, the received signal at the j th MS becomes

$$\mathbf{y}_{n,j} = \mathbf{H}_{n,j} \mathbf{B}_{n,\mu} \sum_{\mu \in \zeta_n} \phi_{n,\mu} + \mathbf{n}_{n,j} = \mathbf{H}_{n,j} \mathbf{B}_{n,\mu} x_n + \mathbf{n}_{n,j} . \tag{2}$$

where $\mathbf{B}_{n,\mu}$ is the precoding matrix with the μ th column equal to $\mathbf{b}_{n,\mu}$ on the n th PRB, $\mu \in \zeta_n$.

With a linear Minimum Mean Square Error (MMSE) receiver, also known as a Wiener filter, the optimum precoding matrix under the sum power constraint can be generally expressed as $\mathbf{B}_{n,j} = \mathbf{U}_{n,j} \sqrt{\mathbf{Ó}_{n,j}} \mathbf{V}_{n,j}$ [9]. Here $\mathbf{U}_{n,j}$ is an $n_t \times n_t$ eigenvector matrix with columns corresponding to the n_t largest eigenvalues of the matrix $\mathbf{H}^0_{n,j} \mathbf{H}^{0\,H}_{n,j}$, where $\mathbf{H}^{0\,H}_{n,j}$ is the Hermitian transpose of the quanized channel matrix $\mathbf{H}^0_{n,j}$. For Schur-Concave objective functions, $\mathbf{V}_{n,j} \in \mathbf{C}^{n_t \times n_t}$ is an unitary matrix, and $\mathbf{Ó}_{n,j}$ is a diagonal matrix with the μ th diagonal entry $\mathbf{Ó}_{n,j}(\eta,\eta)$ representing the power allocated to the η th established data sub-stream, $\eta \in \{1,2,...,n_t\}$.

3. The average channel capacity

The average channel capacity[10] or the so called Shannon (Ergodic) Capacity[11] per PRB can be obtained by

$$C = \int_0^\infty \log_2(1+\gamma) f_\Gamma(\gamma) d\gamma . \tag{3}$$

where $f_\Gamma(\gamma)$ is the probability density function (PDF) of γ, the effective signal to interference plus noise ratio (SINR).

With the investigated linear receivers, which decompose the MIMO channel into independent channels, the total capacity for the multiple input sub-stream MIMO systems is equal to the sum of the capacities for each sub-stream, i.e.,

$$C_{total} = \sum_{i=0}^{Q-1} \int_0^\infty \log_2(1+\gamma) f_{\Gamma_i}(\gamma) d\gamma . \tag{4}$$

where Q is the number of sub-streams.

4. Conclusion

In this paper, we analyzed the average channel capacity of the LTE downlink multiuser systems with linearly precoded SDM MIMO schemes in conjunction with a base station packet scheduler. Both SU and MU MIMO schemes with FDPS are in-vestigated. Our analysis reveals that the system using a linearly precoded MU-MIMO scheme has a larger capacity than the one using a SU-MIMO scheme. For a SU-MIMO scheme, the precoded MIMO system always has a higher average channel capacity than the one without precoding. For a MU-MIMO, the above conclusion does not hold, particularly for systems with a large number of active users.

The analysis conducted in this paper provides a theoretical reference for the practical implementation of the LTE systems.

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